Electrons and phonons in twisted bilayer graphene

F. Guinea

Outline

- Long range interactions in twisted bilayer graphene.
- Bands and Fermi Surface pinning.
- Broken symmetry phases.
- Long range interactions and superconductivity.

instituto IMdea NANOCIENCIA

EXCELENCIA SEVERO OCHOA

DIPC Donostia International Physics Center

In colaboration with N. R. Walet, R. Brown, (U. Manchester), T. Cea, P. Pantaleon, (Imdea). Also V. P. Phong (U. Penn), V. Crépel, L. Fu, L. Levitov (MIT), S. Yuan (Wuhan U.), J. Lischner, Z. Goodwin (Imperial C.).

Superconductivity in graphene

Superconductivity in graphene. March Meeting, Los Angeles 2018

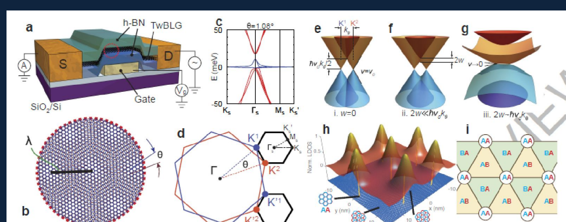
nature Accelerated Article Preview

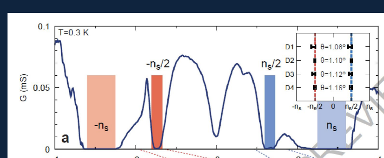
LETTER

doi:10.1038/nature26154

Correlated insulator behaviour at half-filling in magic-angle graphene superlattices

Yuan Cao, Valla Fatemi, Ahmet Demir, Shiang Fang, Spencer L. Tomarken, Jason Y. Luo, J. D. Sanchez-Yamagishi, K. Watanabe, T. Taniguchi, E. Kaxiras, R. C. Ashoori & P. Jarillo-Herrero





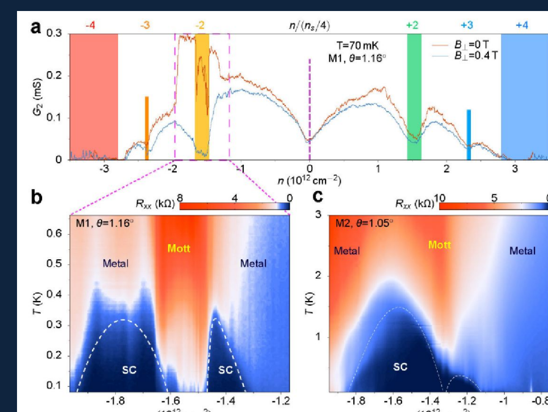
nature Accelerated Article Preview

ARTICLE

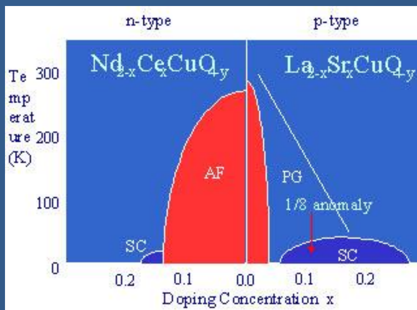
doi:10.1038/nature26160

Unconventional superconductivity in magic-angle graphene superlattices

Yuan Cao, Valla Fatemi, Shiang Fang, Kenji Watanabe, Takashi Taniguchi, Efthimios Kaxiras & Pablo Jarillo-Herrero



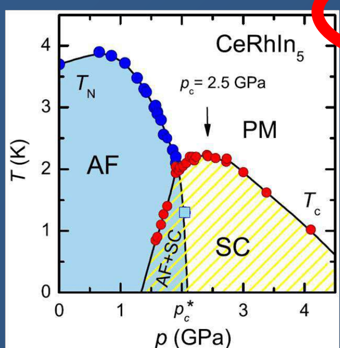
Strongly correlated systems



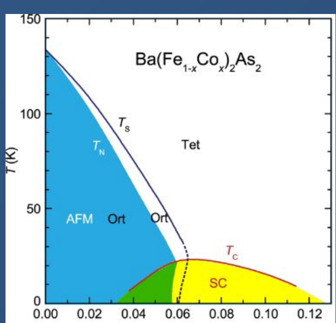
Cuprate superconductors

Pairing due to magnetic fluctuations
Variations of the Hubbard model

?



Heavy fermion compounds



Pnictides

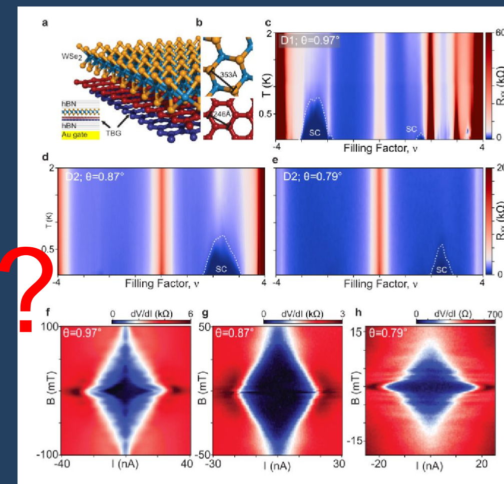
Article

Superconductivity in metallic twisted bilayer graphene stabilized by WSe_2

https://doi.org/10.1038/s41586-020-2473-8
Received: 31 January 2020

Harpreet Singh Arora[1,2,8], Robert Polski[1,2,8], Yiran Zhang[1,2,3,8], Alex Thomson[2,3,4], Youngjoon Choi[1,2,3], Hyunjin Kim[1,2,3], Zhong Lin[5], Ilham Zaky Wilson[5], Xiaodong Xu[5,6], Jiun-Haw Chu[5], Kenji Watanabe[7], Takashi Taniguchi[7], Jason Alicea[2,3,4] & Stevan Nadj-Perge[1,2]

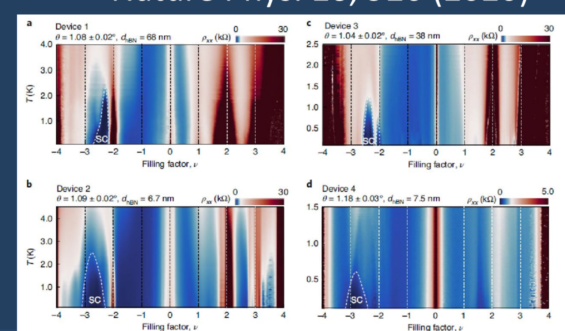
Nature **583**, 579 (2020)



Independent superconductors and correlated insulators in twisted bilayer graphene

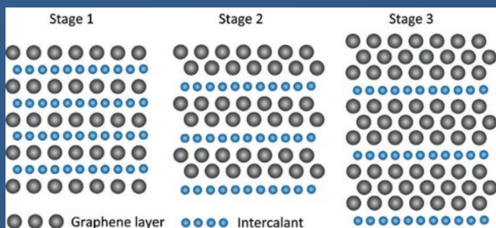
Yu Saito[1], Jingyuan Ge[2], Kenji Watanabe[3], Takashi Taniguchi[3] and Andrea F. Young[2]

Nature Phys. **16**, 926 (2020)

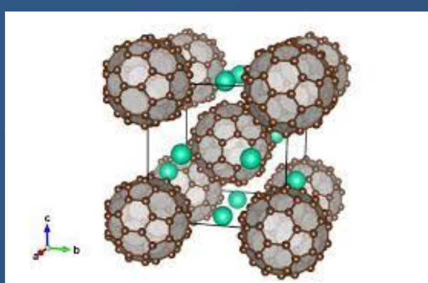


Superconductivity and strongly correlated phases in carbon compunds

Superconductivity



Graphite intercalation compounds, such as C_6Ca, $n \geq 10^{14}\ \mathrm{cm}^{-2}, T_c \sim 1-10\mathrm{K}$



Alkali intercalated fullerenes, such as Rb_3C_{60}, $T_c \sim 10-40\mathrm{K}$

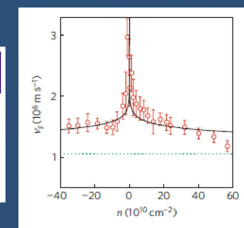
Renormalization of the Fermi velocity

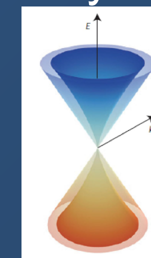
nature physics LETTERS

PUBLISHED ONLINE: 24 JULY 2011 | DOI: 10.1038/NPHYS2049

Dirac cones reshaped by interaction effects in suspended graphene

D. C. Elias[1], R. V. Gorbachev[1], A. S. Mayorov[1], S. V. Morozov[2], A. A. Zhukov[3], P. Blake[3], L. A. Ponomarenko[1], I. V. Grigorieva[1], K. S. Novoselov[1], F. Guinea[4]* and A. K. Geim[1,3]



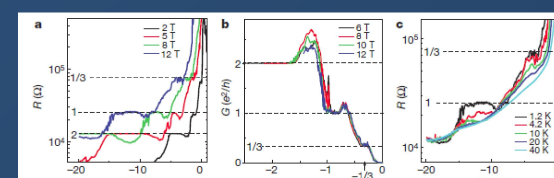


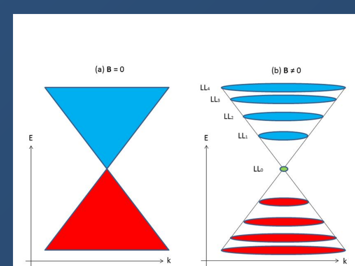
Landau levels

Fractional quantum Hall effect and insulating phase of Dirac electrons in graphene

Xu Du[1]†, Ivan Skachko[1], Fabian Duerr[1], Adina Luican[1] & Eva Y. Andrei[1]

Nature **462**, 192 (2009)

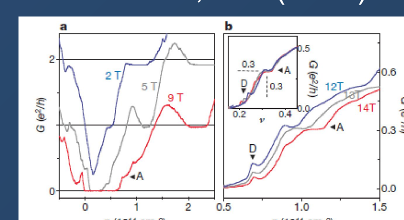




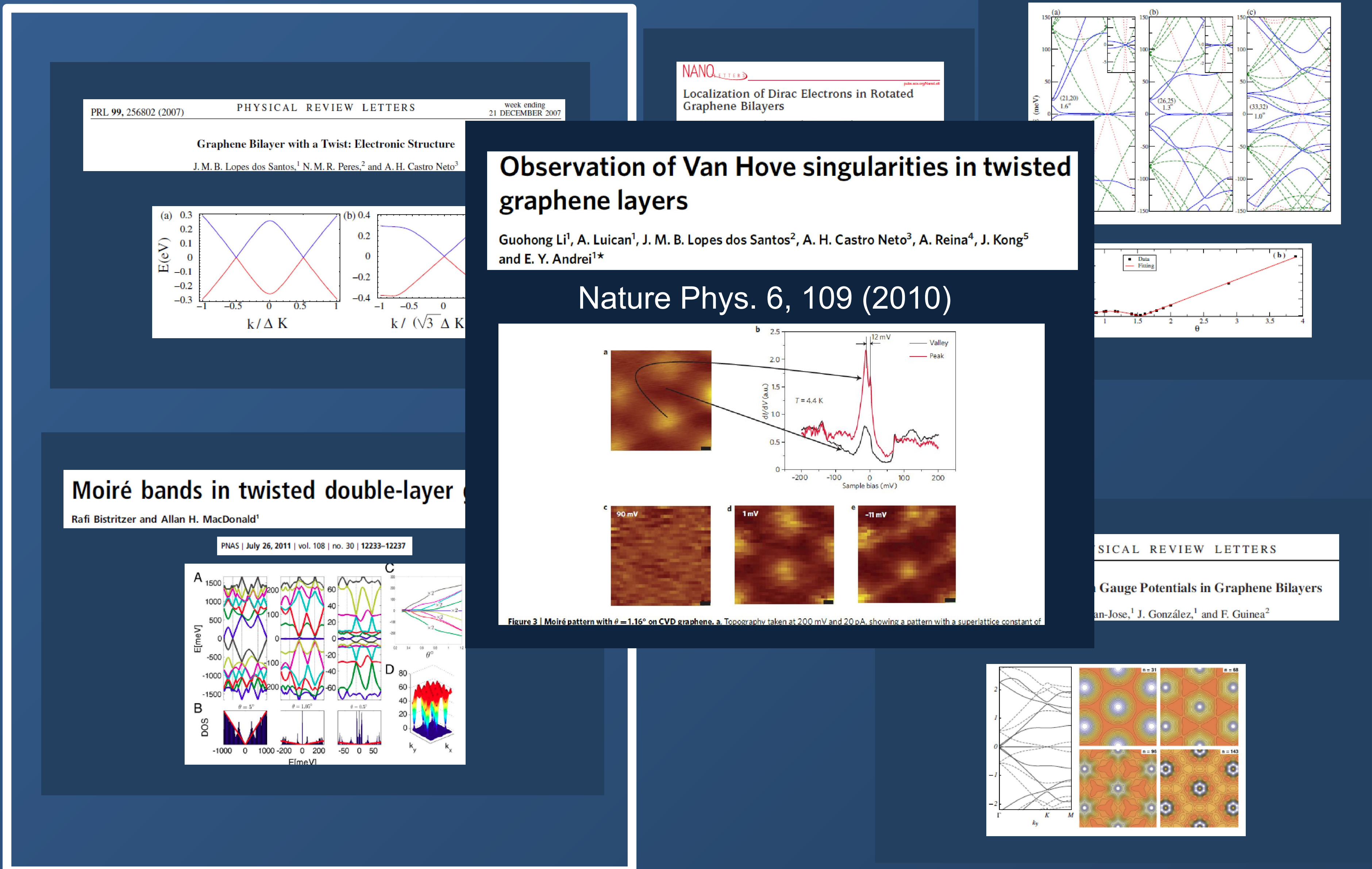
Observation of the fractional quantum Hall effect in graphene

Kirill I. Bolotin[1]*†, Fereshte Ghahari[1]*, Michael D. Shulman[2], Horst L. Stormer[1,2] & Philip Kim[1,2]

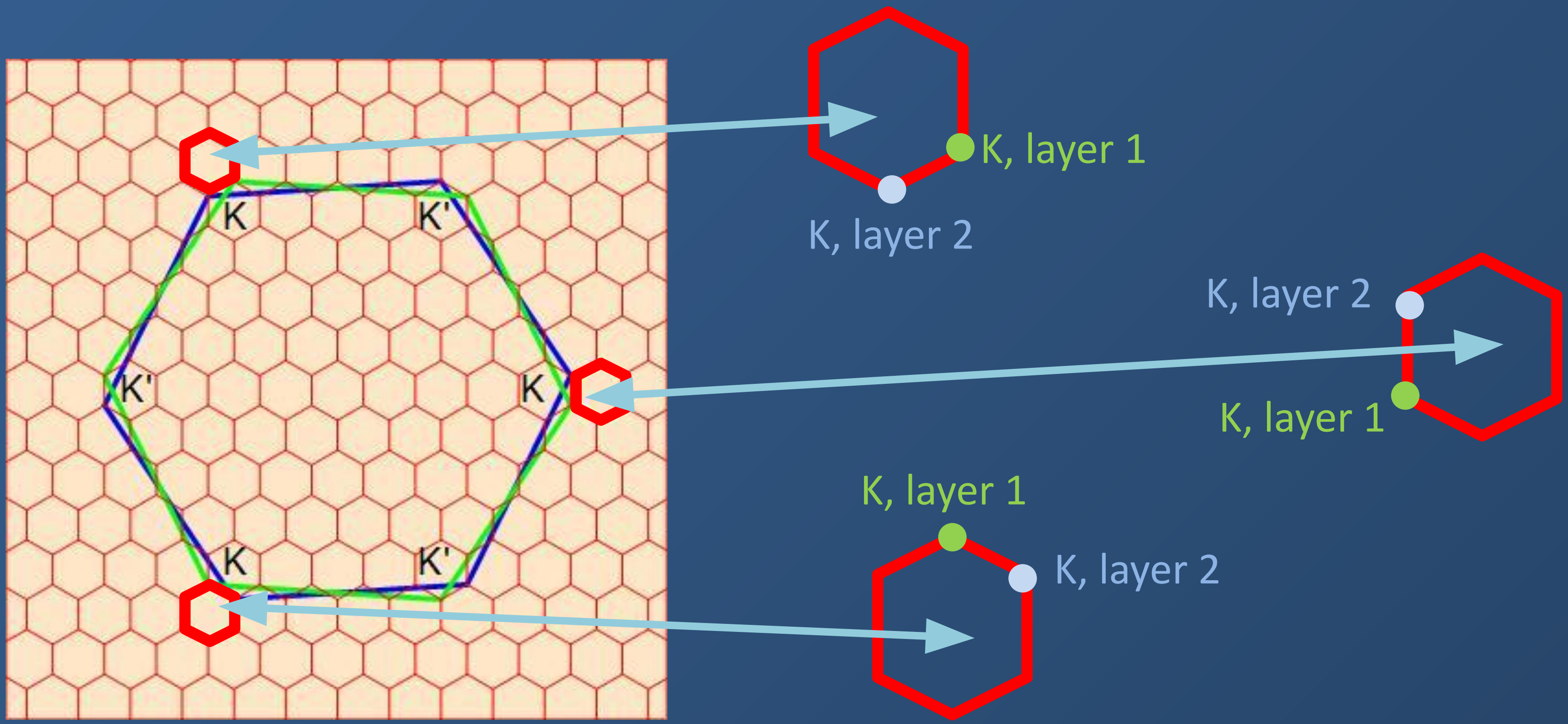
Nature **462**, 196 (2009)



Twisted graphene layers: theory

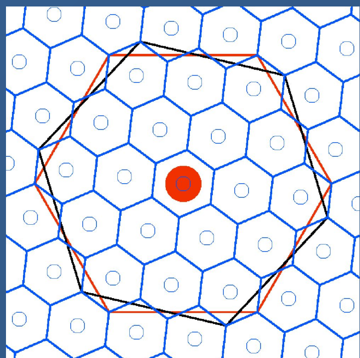


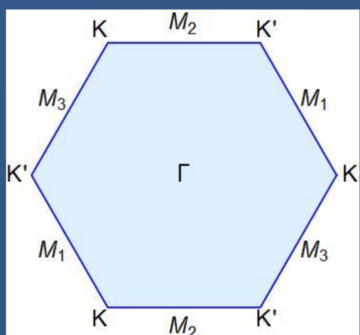
Twisted bilayer graphene: Bloch waves



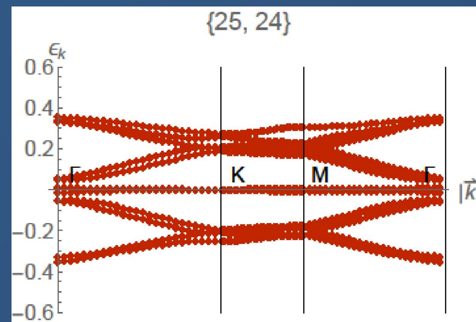
- Wavefunctions at the Dirac points have an internal structure, defined by phases which change at the atomic scale.
- The same wavefunction in one layer is projected onto different wavefunctions in the other layer, depending on the choice of phases.

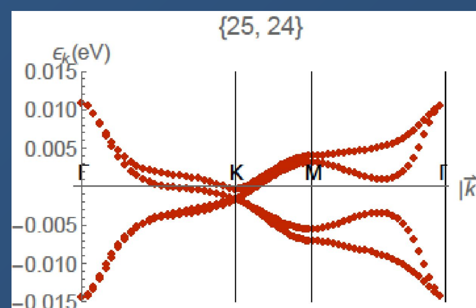
Electronic structure



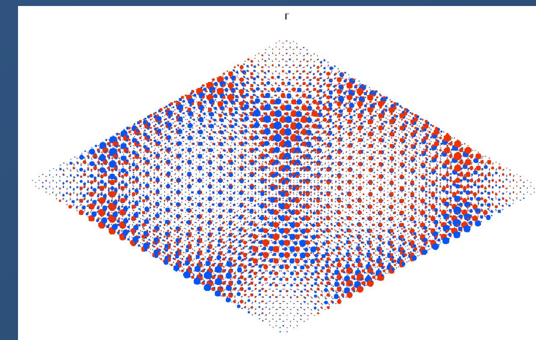


Brillouin zones

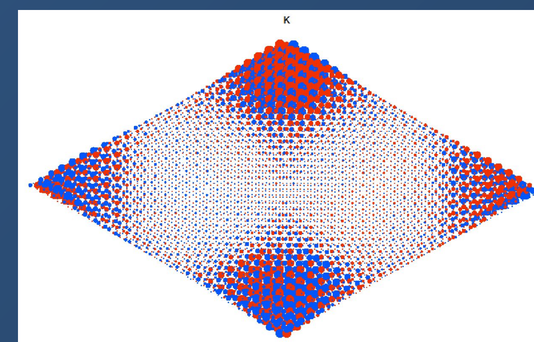




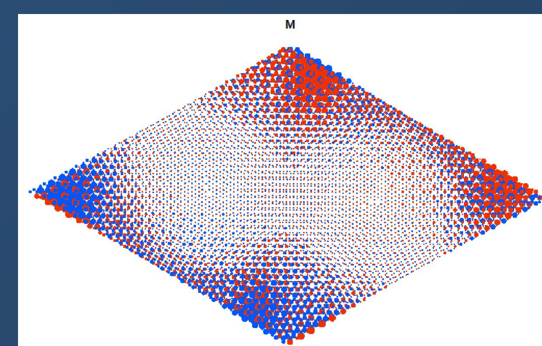
Low energy bands



Γ

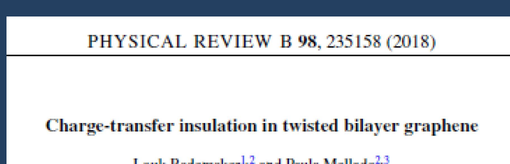


K



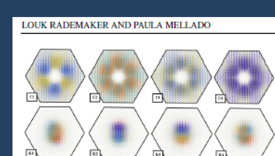
M

Charge density distribution


PHYSICAL REVIEW B 98, 235158 (2018)

Charge-transfer insulation in twisted bilayer graphene

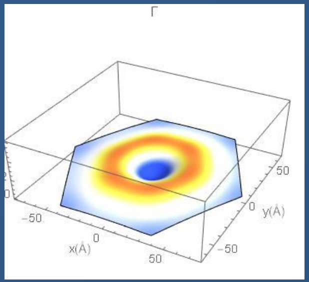
Louk Rademaker[1,2] and Paula Mellado[2,3]

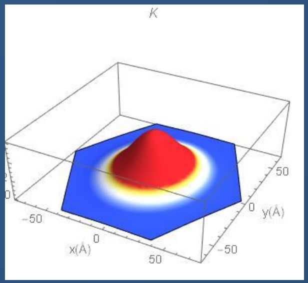


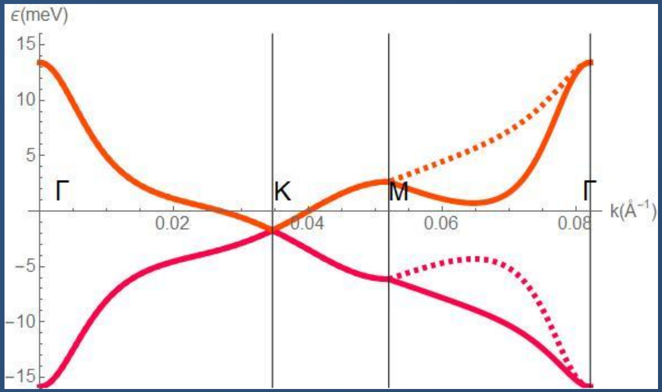
Magic angles and beyond

angle $= 1.05^0$

Γ K

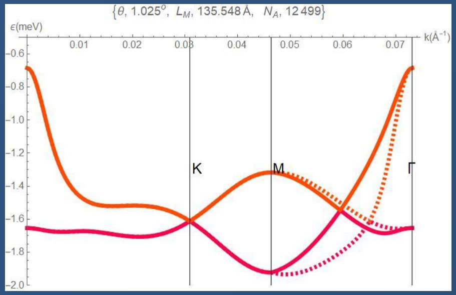


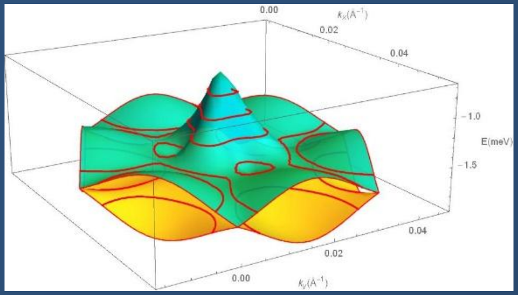


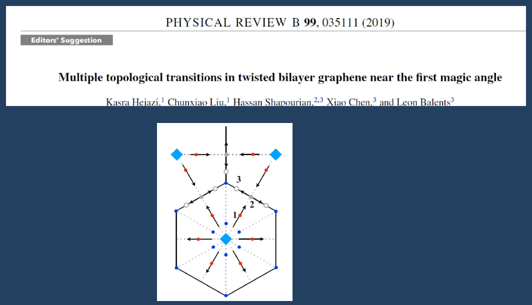


Different charge distributions at different points in the Brillouin Zone

angle $= 1.025^0$



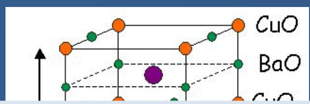




- Additional Dirac points
- Bifurcation of van Hove singularities

Origin of narrow bands

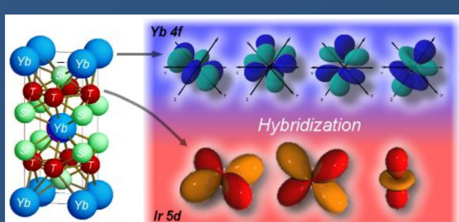
Localized orbitals and complex unit cells



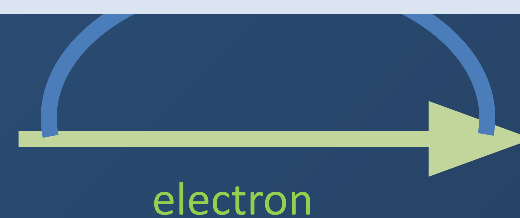
Quasiparticles dressed by excitations

Twisted bilayer graphene

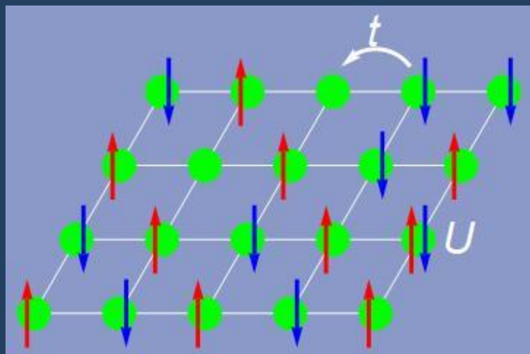
- No interactions are included
- A plane wave in one layer is transferred to the other layer as a superposition of three waves.
- The hamiltonian is defined by one, or two, dimensionless parameters.
- The energy scales are of order 100 meV



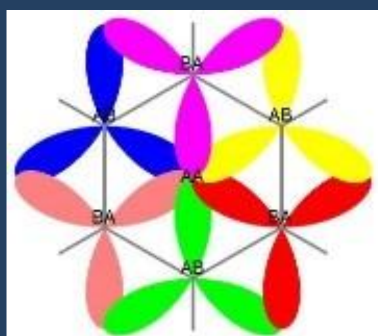
Heavy fermion materials. Compounds with rare earths



(Some) theoretical models

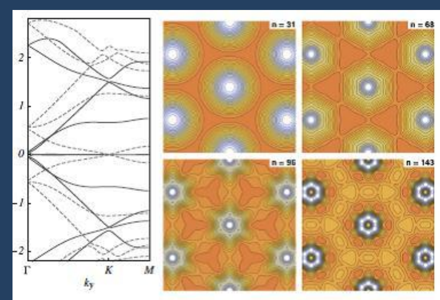


Hubbard model

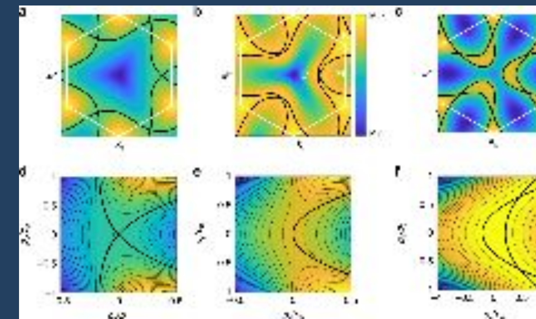


Wannier functions

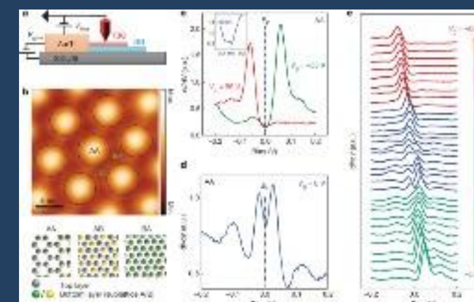
Landau levels



Chiral model



Van Hove singularities



Peaks in the density of states

PHYSICAL REVIEW LETTERS 123, 087602 (2019)

pin-Orbital Density Wave and a Mott Insulator in a Two-Orbital Hubbard Model on a Honeycomb Lattice

Zheng Zhu,[1,2,3] D. N. Sheng,[2,*] and Liang Fu[1,†]

PHYSICAL REVIEW B 98, 245103 (2018)

Correlations and electronic order in a two-orbital honeycomb lattice model for twisted bilayer graphene

Jörn W. F. Venderbos[1,2] and Rafael M. Fernandes[3]

PHYSICAL REVIEW RESEARCH 2, 023238 (2020)

Chern bands of twisted bilayer graphene: Fractional Chern insulators and spin phase transition

Cécile Repellin[1,2] and T. Senthil[2]

PHYSICAL REVIEW RESEARCH 2, 023237 (2020)

Fractional Chern insulator states in twisted bilayer graphene: An analytical approach

Patrick J. Ledwith, Grigory Tarnopolsky, Eslam Khalaf, and Ashvin Vishwanath

PHYSICAL REVIEW LETTERS 122, 026801 (2019)

Kohn-Luttinger Superconductivity in Twisted Bilayer Graphene

J. González[1] and T. Stauber[2]

PHYSICAL REVIEW B 101, 224513 (2020)

Editors' Suggestion

Nematic superconductivity in twisted bilayer graphene

Dmitry V. Chichinadze, Laura Classen, and Andrey V. Chubukov

Local orbitals and Wannier functions

PHYSICAL REVIEW B 98, 045103 (2018)

Editors' Suggestion

Model for the metal-insulator transition in graphene superlattices and beyond

Noah F. Q. Yuan and Liang Fu

PHYSICAL REVIEW X 8, 031087 (2018)

Maximally Localized Wannier Orbitals and the Extended Hubbard Model for Twisted Bilayer Graphene

Mikito Koshino,[1,*] Noah F. Q. Yuan,[2] Takashi Koretsune,[3] Masayuki Ochi,[1] Kazuhiko Kuroki,[1] and Liang Fu[2]

PHYSICAL REVIEW X 8, 031088 (2018)

Symmetry, Maximally Localized Wannier States, and a Low-Energy Model for Twisted Bilayer Graphene Narrow Bands

Jian Kang[1,*] and Oskar Vafek[1,2,†]

PHYSICAL REVIEW X 8, 031089 (2018)

Origin of Mott Insulating Behavior and Superconductivity in Twisted Bilayer Graphene

Hoi Chun Po,[1] Liujun Zou,[1,2] Ashvin Vishwanath,[1] and T. Senthil[2]

PHYSICAL REVIEW B 98, 085435 (2018)

Editors' Suggestion

Band structure of twisted bilayer graphene: Emergent symmetries, commensurate approximants, and Wannier obstructions

Liujun Zou,[1,2] Hoi Chun Po,[1] Ashvin Vishwanath,[1] and T. Senthil[2]

Electronic bands of twisted graphene layers

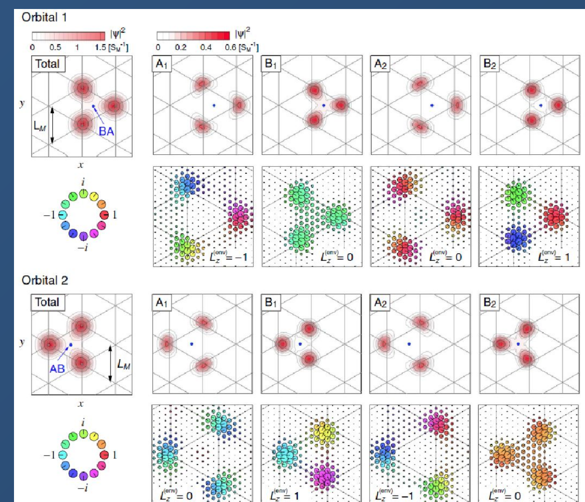
1. Model for Metal-Insulator Transition in Graphene Superlattices and Beyond
Authors: Noah F. Q. Yuan, Liang Fu
arXiv:1803.09699, Phys. Rev. B 98, 079901 (2018)
2. Origin of Mott Insulating Behavior and Superconductivity in Twisted Bilayer Graphene
Authors: Hoi Chun Po, Liujun Zou, Ashvin Vishwanath, and T. Senthil
arXiv:1803.09742, Phys. Rev. X 8, 031089 (2018)
3. Symmetry, Maximally Localized Wannier States, and a Low-Energy Model for Twisted Bilayer Graphene Narrow Bands
Authors: Jian Kang and Oskar Vafek
arXiv:1805.04918, Phys. Rev. X 8, 031088 (2018)
4. Maximally-localized Wannier orbitals and the extended Hubbard model for the twisted bilay...
Authors: Mikito Koshi...
Kazuhiko Kuroki, Lian...
arXiv:1805.06819, Phys...
5. Band Structure of T...
Commensurate App...
Authors: Liujun Zou, H...
arXiv:1806.07873, Phys. Rev. B 98, 085435 (2018)

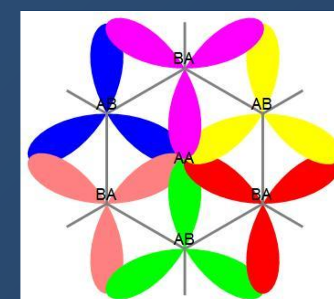
Recommended with a Commentary by Francisco Guinea, Imdea

Journal Club for Condensed Matter Physics

A Monthly Selection of Interesting Papers by Distinguished Correspondents

- The underlying structure of the superlattice is a honeycomb lattice.
- The lattice nodes are at the centers of the regions where the stacking is AB aor BA.
- The Wannier functions have maxima at three lobes around the nodes, and non trivial phases.





This description differs significantly from an array of mesoscopic quantum dots in a triangular lattice.

https://www.condmatjclub.org

Local orbitals and Wannier functions. Fragile topology

REVIEWS OF MODERN PHYSICS, VOLUME 84, OCTOBER–DECEMBER 2012

Maximally localized Wannier functions: Theory and applications

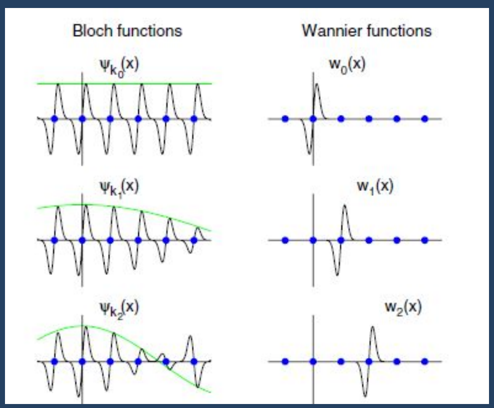
Nicola Marzari
Theory and Simulation of Materials (THEOS), École Polytechnique Fédérale de Lausanne, Station 12, 1015 Lausanne, Switzerland

Arash A. Mostofi
Departments of Materials and Physics, and the Thomas Young Centre for Theory and Simulation of Materials, Imperial College London, London SW7 2AZ, United Kingdom

Jonathan R. Yates
Department of Materials, University of Oxford, Parks Road, Oxford OX1 3PH, United Kingdom

Ivo Souza
Centro de Física de Materiales (CSIC) and DIPC, Universidad del País Vasco, 20018 San Sebastián, Spain and Ikerbasque Foundation, 48011 Bilbao, Spain

David Vanderbilt
Department of Physics and Astronomy, Rutgers University, Piscataway, New Jersey 08854-8019, USA



The Wannier functions are not uniquely defined

$$\Psi(\vec{r}) \propto \int e^{i\phi_{\vec{k}}} \psi_{\vec{k}} \, d^2\vec{k}$$

PHYSICAL REVIEW B **74**, 235111 (2006)

Insulator/Chern-insulator transition in the Haldane model

T. Thonhauser and David Vanderbilt
Department of Physics and Astronomy, Rutgers, The State University of New Jersey, Piscataway, New Jersey 08854, USA

(Received 24 August 2006; published 20 December 2006)

We study the behavior of several physical properties of the Haldane model as the system undergoes its transition from the normal-insulator to the Chern-insulator phase. We find that the density matrix has expo- [illegible] phases, while having a power-law decay, more characteristic of a metallic system, precisely at the phase boundary. [illegible]otal spread of the maximally localized Wannier functions is found to diverge in the Chern-insulator phase. [illegible]owever, its gauge-invariant part, related to the localization length of Resta and Sorella, is finite in [illegible]nsulating phases and diverges as the phase boundary is ap- [illegible] usual algorithms for constructing Wannier functions break down as one crosses into the Chern-insulator region of the phase diagram.

Singularities in the Berry phase prevent the existence of exponentially localized Wannier functions.

The “obstruction"

PHYSICAL REVIEW LETTERS **121**, 126402 (2018)

Fragile Topology and Wannier Obstructions

Hoi Chun Po,[1] Haruki Watanabe,[2] and Ashvin Vishwanath[1]

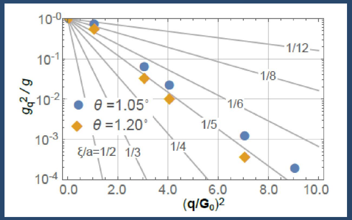
$$\lim_{|\vec{r}^2| \to \infty} w(\vec{r}) \propto \frac{1}{r^{3/2}}$$

$$\langle w(\vec{r}) | \vec{r}^2 | w(\vec{r}) \rangle \to \infty$$

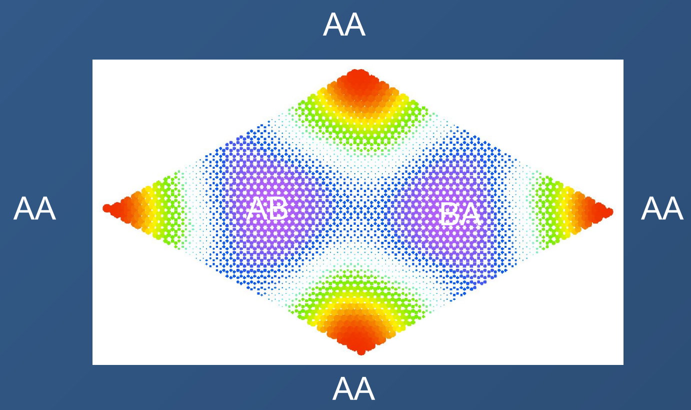
Tunable electron-phonon interactions in long-period superlattices

Hiroaki Ishizuka,[1,2,*] Ali Fahimniya,[1] Francisco Guinea,[3,4] and Leonid Levitov[1]

Nano Lett. **21**, 6475 (2021)

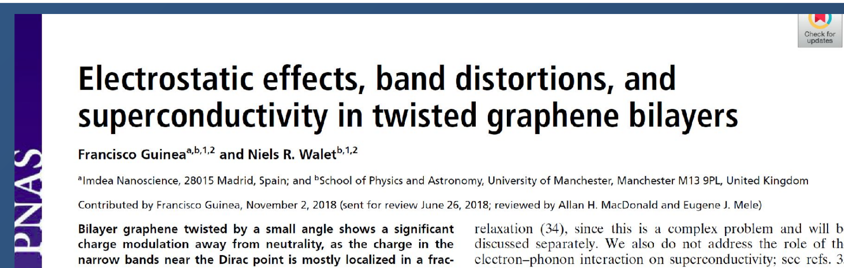


Interactions in twisted bilayer graphene



Number of atoms:		
Moiré unit length:		
Radius of the charge distribution:		
Coulomb energy:		
Intraatomic (Hubbard) repulsion:		
Electron-phonon coupling:		

Coulomb interactions and screening in twisted graphene bilayers



Electrostatic effects, band distortions, and superconductivity in twisted graphene bilayers

Francisco Guinea[a,b,1,2] and Niels R. Walet[b,1,2]

[a]Imdea Nanoscience, 28015 Madrid, Spain; and [b]School of Physics and Astronomy, University of Manchester, Manchester M13 9PL, United Kingdom

Contributed by Francisco Guinea, November 2, 2018 (sent for review June 26, 2018; reviewed by Allan H. MacDonald and Eugene J. Mele)

Bilayer graphene twisted by a small angle shows a significant charge modulation away from neutrality, as the charge in the narrow bands near the Dirac point is mostly localized in a frac-

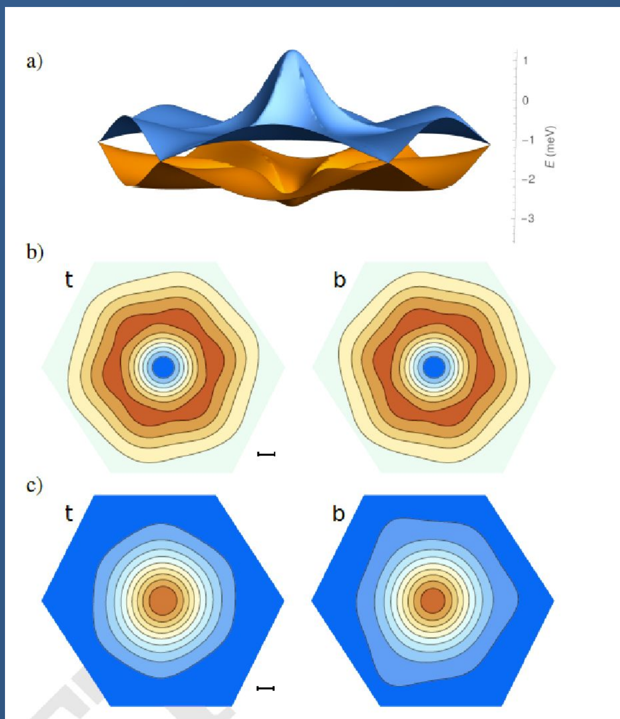
relaxation (34), since this is a complex problem and will be discussed separately. We also do not address the role of the electron–phonon interaction on superconductivity; see refs. 35

arXiv:1806.05990

Proc. Nat. Acad. Sci. (USA) **115**, 13174 (2019)

Angle: $\theta = 1.05^o$

Moiré unit cell: $L_M \approx 15\text{nm}$

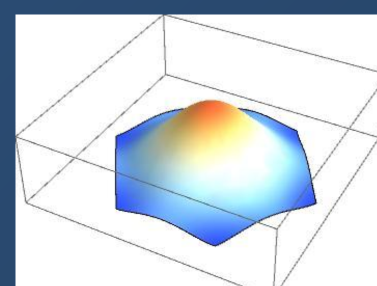


Γ point

K point

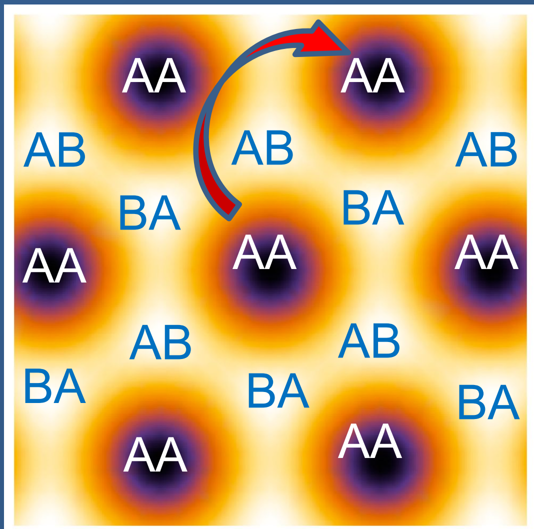
- The charge distribution within the Moiré unit cell depends on the state.
- Away from the neutrality point, the charge is concentrated at the center of the unit cell.
- A non uniform electrostatic potential is induced.

Bands, wavefunctions



Sketch of the electrostatic potential

New interactions in twisted bilayers



- Electron assisted hopping
- Assisted hopping term due to electrostatic interactions
- Favorable for superconductivity

PROCEEDINGS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

On the Electron Theory of Metals

S. Schubin and S. Wonsowsky

Proc. R. Soc. Lond. A 1934 **145**, doi: 10.1098/rspa.1934.0089, published 2 June 1934

J. Phys. C: Solid State Phys., Vol. 12, 1979. Printed in Great Britain. © 1979

Some types of instabilities in the electron energy spectrum of the polar model of the crystal: II. The criterion of stability of a metallic state

PHYSICAL REVIEW B VOLUME 41, NUMBER 10 1 APRIL 1990

Hole superconductivity and the high-T_c oxides

F. Marsiglio and J. E. Hirsch

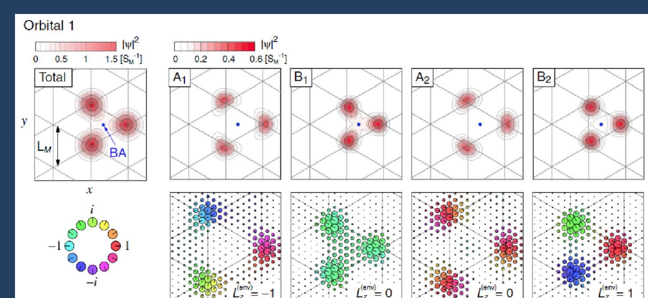
41 6435 ©1990 The American Physical Society

$$\tilde{t} \sum \left(c_i^\dagger c_j + c_j^\dagger c_i\right)\left(n_i + n_j\right) \qquad \tilde{t} \approx V_H$$

PHYSICAL REVIEW X 8, 031087 (2018)

Maximally Localized Wannier Orbitals and the Extended Hubbard Model for Twisted Bilayer Graphene

Mikito Koshino,[1,*] Noah F. Q. Yuan,[2] Takashi Koretsune,[3] Masayuki Ochi,[1] Kazuhiko Kuroki,[1] and Liang Fu[2]



See also

PHYSICAL REVIEW LETTERS 122, 246401 (2019)

Strong Coupling Phases of Partially Filled Twisted Bilayer Graphene Narrow Bands

Jian Kang[1,*] and Oskar Vafek[1,2,†]

PHYSICAL REVIEW LETTERS 122, 246402 (2019)

Ferromagnetic Mott state in Twisted Graphene Bilayers at the Magic Angle

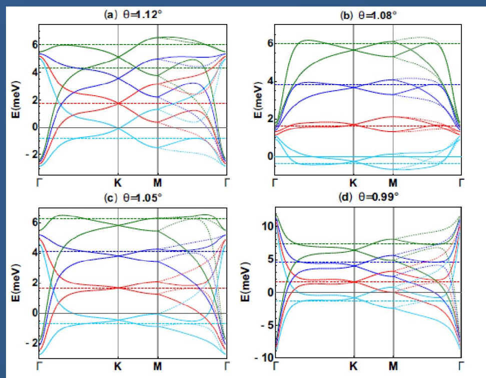
Kangjun Seo,[1] Valeri N. Kotov,[2] and Bruno Uchoa[1,*]

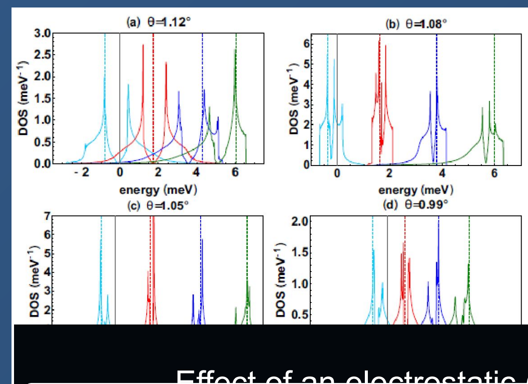
Shapes and widths of the bands: theory

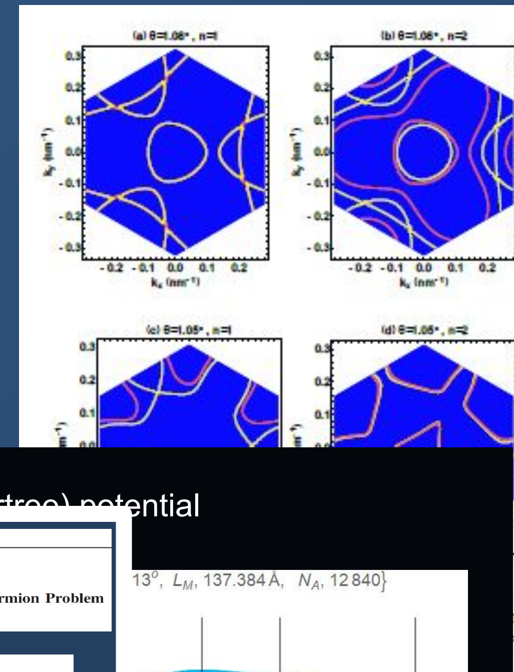
PHYSICAL REVIEW B 100, 205113 (2019)

Electronic band structure and pinning of Fermi energy to Van Hove singularities in twisted bilayer graphene: A self-consistent approach

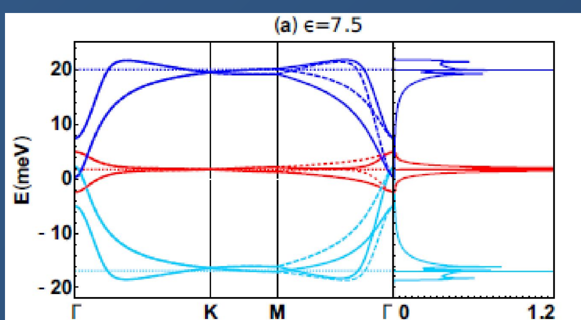
Tommaso Cea,[1,*] Niels R. Walet,[2,†] and Francisco Guinea[1,2,‡]







$$V_H(AA) - V_H(AB) \approx 5\text{meV}$$

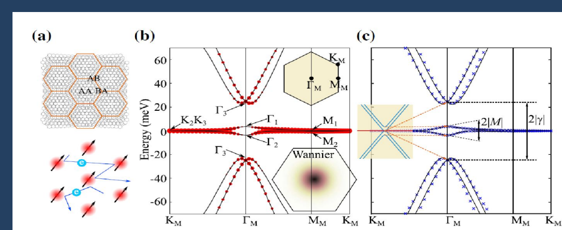


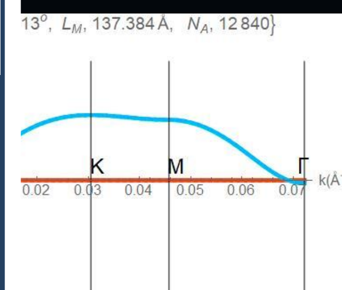
Effect of an electrostatic (Hartree) potential

PHYSICAL REVIEW LETTERS 129, 047601 (2022)
Editors' Suggestion
Featured in Physics

Magic-Angle Twisted Bilayer Graphene as a Topological Heavy Fermion Problem

Zhi-Da Song[1,2,*] and B. Andrei Bernevig[2,3,4]





Chiral model

shifted, while the center, Γ, is not.

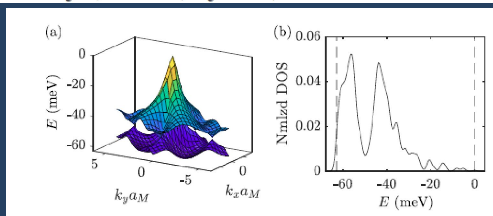
- The bandwidth is increased, although the density of states at the Fermi energy remains high (pinning of van Hove singularities).

Shapes and widths of the bands: theory

PHYSICAL REVIEW RESEARCH **3**, 013033 (2021)

Nematic topological semimetal and insulator in magic-angle bilayer graphene at charge neutrality

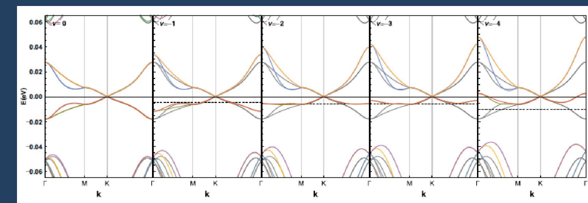
Shang Liu, Eslam Khalaf, Jong Yeon Lee, and Ashvin Vishwanath



PHYSICAL REVIEW B **100**, 205114 (2019)

Charge smoothening and band flattening due to Hartree corrections in twisted bilayer graphene

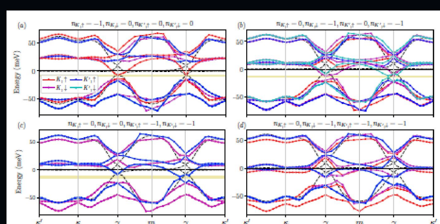
Louk Rademaker,[1] Dmitry A. Abanin,[1] and Paula Mellado[2]



PHYSICAL REVIEW LETTERS **124**, 097601 (2020)

Nature of the Correlated Insulator States in Twisted Bilayer Graphene

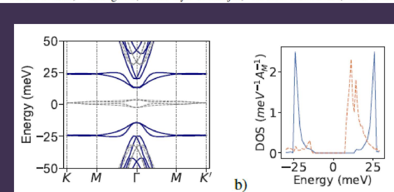
Ming Xie and A. H. MacDonald



PHYSICAL REVIEW X **10**, 031034 (2020)

Ground State and Hidden Symmetry of Magic-Angle Graphene at Even Integer Filling

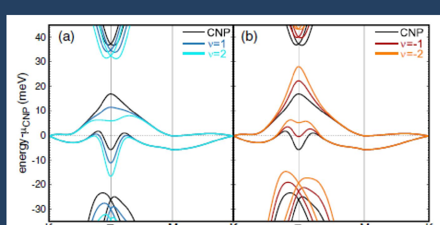
Nick Bultinck,[1,†] Eslam Khalaf,[2,†] Shang Liu,[2] Shubhayu Chatterjee,[1] Ashvin Vishwanath,[2] and Michael P. Zaletel[1,*]



PHYSICAL REVIEW B **102**, 155149 (2020)

Interactions in the 8-orbital model for twisted bilayer graphene

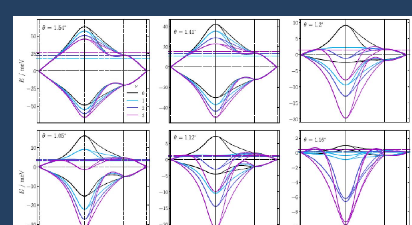
M. J. Calderón* and E. Bascones†



Hartree theory calculations of quasiparticle properties in twisted bilayer graphene

Zachary A H Goodwin, Valerio Vitale, Xia Liang, Arash A Mostofi and Johannes Lischner[1]

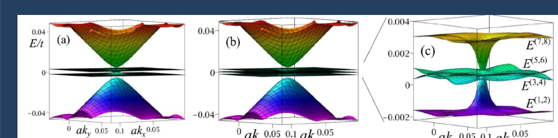
Elec. Struc. **2**, 034001 (2020)



PHYSICAL REVIEW B **100**, 045111 (2019)

Many-body effects in twisted bilayer graphene at low twist angles

A. O. Sboychakov,[1,2] A. V. Rozhkov,[1,2,3,4] A. L. Rakhmanov,[1,2,3,5] and Franco Nori[1,6]

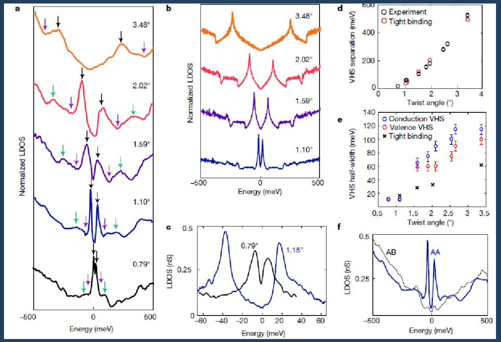


Shapes and widths of the bands: STM experiments

Maximized electron interactions at the magic angle in twisted bilayer graphene

Alexander Kerelsky[1], Leo J. McGilly[1], Dante M. Kennes[2], Lede Xian[3], Matthew Yankowitz[1], Shaowen Chen[1,4], K. Watanabe[5], T. Taniguchi[5], James Hone[6], Cory Dean[1], Angel Rubio[3,7]* & Abhay N. Pasupathy[1]*

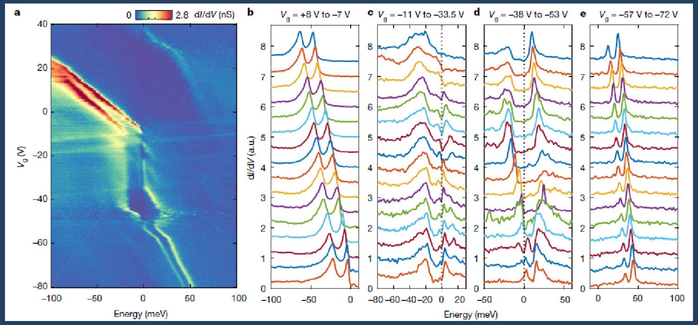
Nature **572**, 95 (2019)



Spectroscopic signatures of many-body correlations in magic-angle twisted bilayer graphene

Yonglong Xie[1], Biao Lian[2], Berthold Jäck[1], Xiaomeng Liu[1], Cheng-Li Chiu[1], Kenji Watanabe[3], Takashi Taniguchi[3], B. Andrei Bernevig[1] & Ali Yazdani[1]*

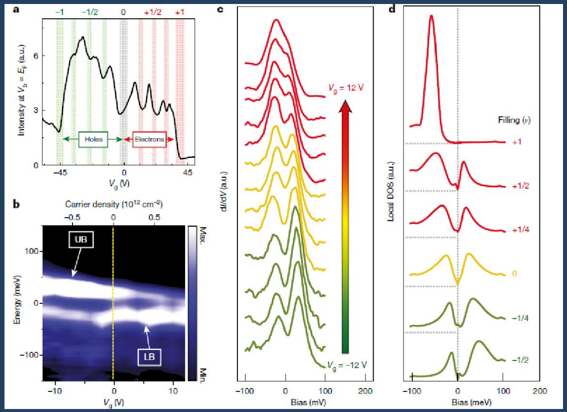
Nature **572**, 101 (2019)



Charge order and broken rotational symmetry in magic-angle twisted bilayer graphene

Yuhang Jiang[1], Xinyuan Lai[1], Kenji Watanabe[2], Takashi Taniguchi[2], Kristjan Haule[1], Jinhai Mao[1,3]* & Eva Y. Andrei[1]*

Nature **572**, 91 (2019)



Article

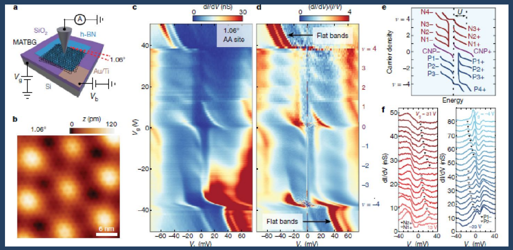
Cascade of electronic transitions in magic-angle twisted bilayer graphene

https://doi.org/10.1038/s41586-020-2339-0

Dillon Wong[1,2,8], Kevin P. Nuckolls[1,2,8], Myungchul Oh[1,2,8], Biao Lian[3,8], Yonglong Xie[1,2,5,6], Sangjun Jeon[1,2,7], Kenji Watanabe[4], Takashi Taniguchi[4], B. Andrei Bernevig[2] & Ali Yazdani[1,2]

Received: 25 November 2019

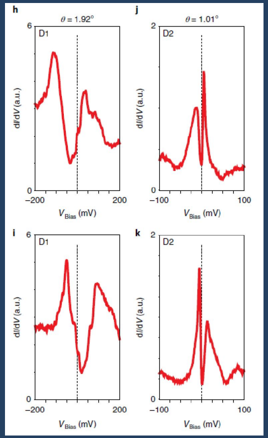
Nature **582**, 198 (2020)



Electronic correlations in twisted bilayer graphene near the magic angle

Youngjoon Choi[1,2,3], Jeannette Kemmer[1,2], Yang Peng[2,3,4], Alex Thomson[2,3,4], Harpreet Arora[1,2], Robert Polski[1,2], Yiran Zhang[1,2,3], Hechen Ren[1,2], Jason Alicea[2,3,4], Gil Refael[2,3,4], Felix von Oppen[2,5], Kenji Watanabe[6], Takashi Taniguchi[6] and Stevan Nadj-Perge[1,2]*

Nature Phys. **15**, 1174 (2019)



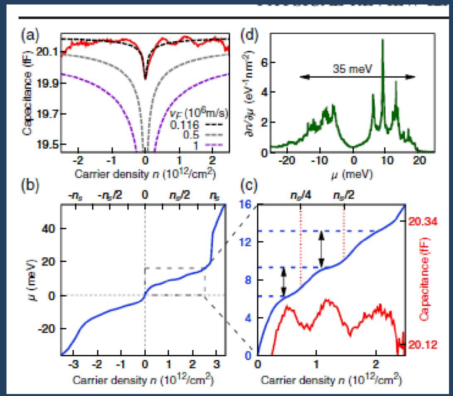
Shapes and widths of the bands: compressibility experiments

PHYSICAL REVIEW LETTERS 123, 046601 (2019)

Editors' Suggestion

Electronic Compressibility of Magic-Angle Graphene Superlattices

S. L. Tomarken,[1] Y. Cao,[1] A. Demir,[1] K. Watanabe,[2] T. Taniguchi,[2] P. Jarillo-Herrero,[1,*] and R. C. Ashoori[1,†]



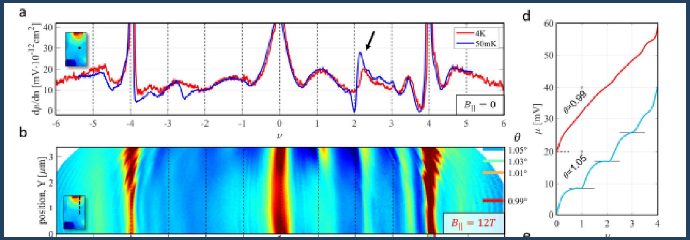
Article

Cascade of phase transitions and Dirac revivals in magic-angle graphene

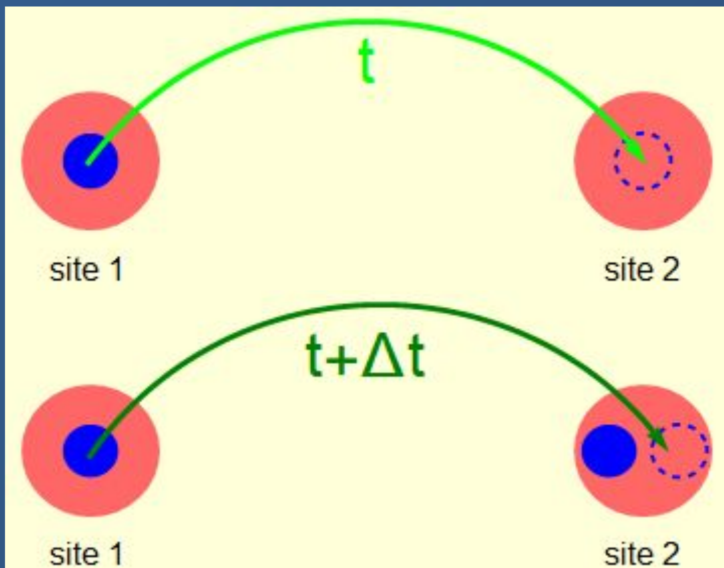
https://doi.org/10.1038/s41586-020-2373-y
Received: 25 November 2019

U. Zondiner[1,5], A. Rozen[1,5], D. Rodan-Legrain[2,5], Y. Cao[2], R. Queiroz[1], T. Taniguchi[3], K. Watanabe[3], Y. Oreg[1], F. von Oppen[4], Ady Stern[1], E. Berg[1], P. Jarillo-Herrero[2] & S. Ilani[1]

Nature **582**, 203 (2020).



Electron assisted hopping



PROCEEDINGS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

On the Electron Theory of Metals

S. Schubin and S. Wonsowsky

Proc. R. Soc. Lond. A 1934 **145**, doi: 10.1098/rspa.1934.0089, published 2 June 1934

J. Phys. C: Solid State Phys., Vol. 12, 1979. Printed in Great Britain. © 1979

Some types of instabilities in the electron energy spectrum of the polar model of the crystal: I. The maximum-polarity state

S V Vonsovsky and M I Katsnelson

PHYSICAL REVIEW B VOLUME 41, NUMBER 10 1 APRIL 1990

Hole superconductivity and the high-T_c oxides

F. Marsiglio and J. E. Hirsch

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Electron assisted hopping, a simple model

SCIENCE ADVANCES | RESEARCH ARTICLE

MATERIALS SCIENCE

New mechanism and exact theory of superconductivity from strong repulsive interaction

Valentin Crépel* and Liang Fu*

Sci. Adv. 2021; 7 : eabh2233 23 July 2021

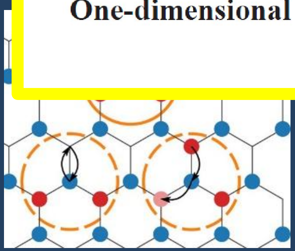
$$\mathcal{H} = -t \sum_{\langle r,r' \rangle} \left(c_r^\dagger c_{r'} + h.c. \right) + \Delta \sum_{r \in B} n_r + V \sum_{\langle r,r' \rangle} n_r n_{r'}$$

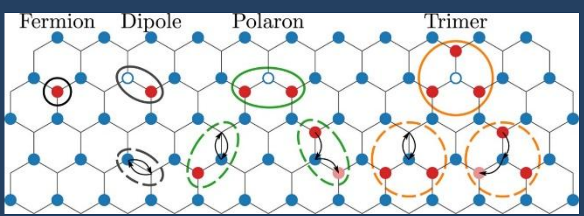
Spinless electrons in the (sublattice polarized) honeycomb la

PHYSICAL REVIEW B, VOLUME 64, 033102

One-dimensional spinless fermion model with competing interactions beyond half filling

A. K. Zhuravlev and M. I. Katsnelson





PHYSICAL REVIEW B 105, 094506 (2022)

Editors' Suggestion | Featured in Physics

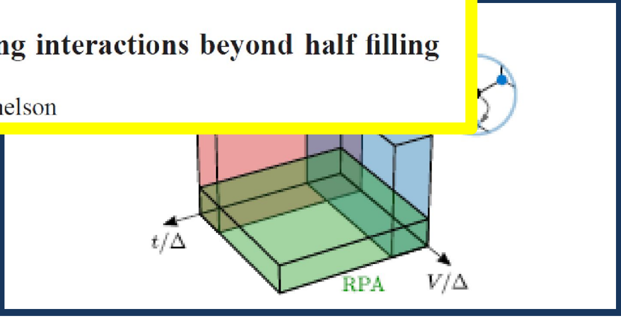
Unconventional superconductivity due to interband polarization

Valentin Crépel,[1] Tommaso Cea,[2] Liang Fu,[1] and Francisco Guinea[2,3,4]

Physics

Explaining Superconductivity in 2D Materials

Interactions between electrons in multiple electronic bands can produce a strong coupling among electrons—providing the "glue" that holds



- Simple model of a superconductor by repulsive interactions
- p-wave
- Finite gap, with opposite signs at the two valleys

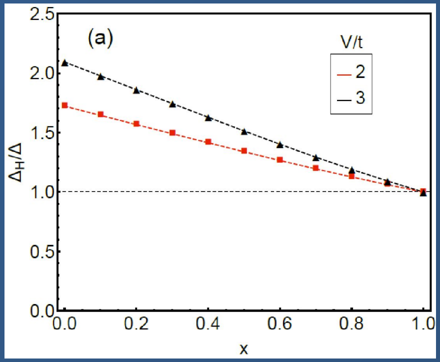
Results

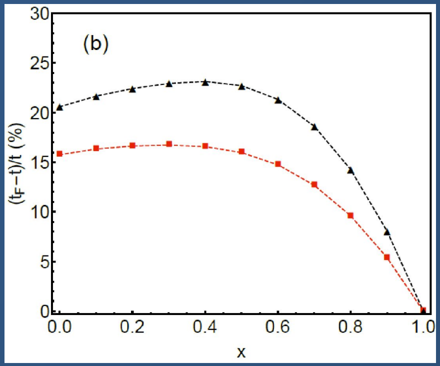
PHYSICAL REVIEW B **105**, 094506 (2022)

Editors' Suggestion | Featured in Physics

Unconventional superconductivity due to interband polarization

Valentin Crépel,[1] Tommaso Cea,[2] Liang Fu,[1] and Francisco Guinea[2,3,4]





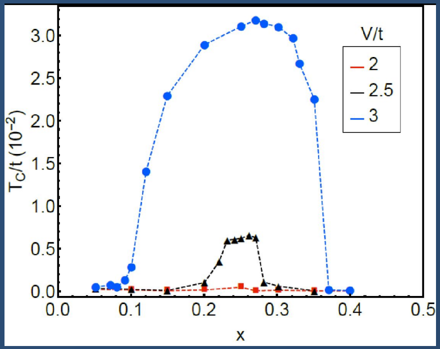
Hartree-Fock

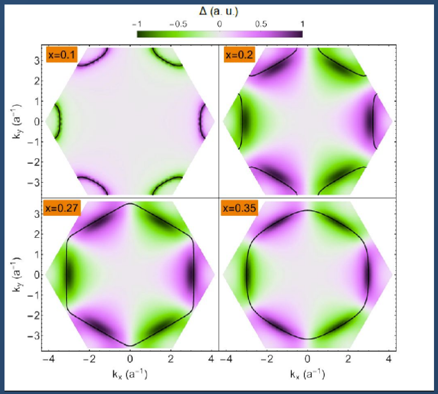
$$\tilde{\mathcal{H}} = \sum_{\vec{k},\sigma,\tau} \frac{|\vec{k}|^2}{2m} \psi^{\dagger}_{\vec{k},\sigma,\tau} \psi_{\vec{k},\sigma,\tau} + \frac{g_0}{N_f} \sum_{\vec{q}} \rho_{\vec{q}} \rho_{-\vec{q}}$$

$$g_0 = -\frac{27t^2V^2}{N_f(\Delta + 3V)^3}$$

- Nodeless superconductivity
- Attraction at small momenta

$\Delta/t=10$

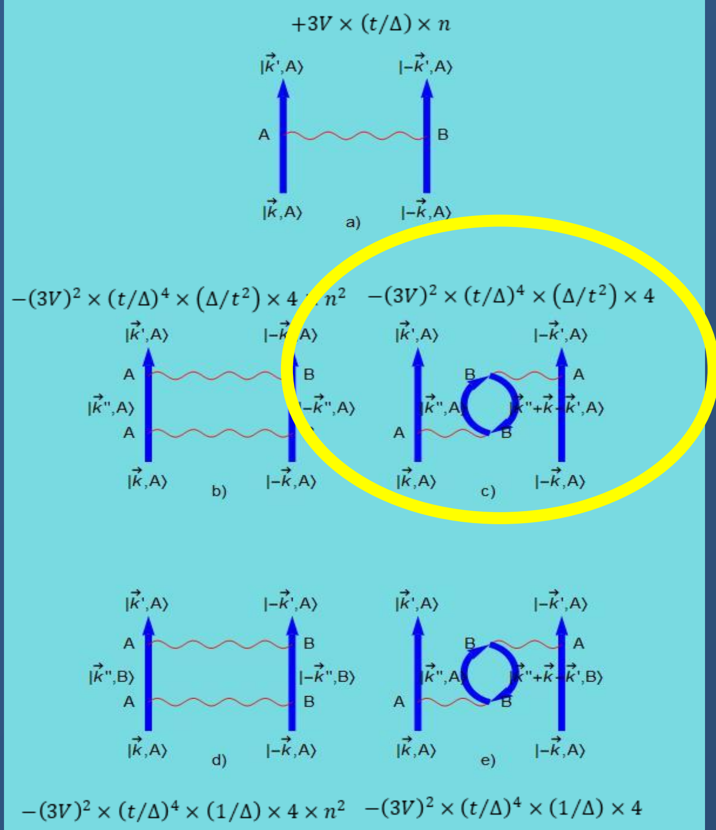




Superconductivity

Attractive interaction for small momenta

Screening gives rise to new attractive interactions



Hartree potential

$$\Delta_H = 3V \sum_{\vec{k},\sigma,\tau \in occ} \left\langle \vec{k},\sigma,\tau \middle| \sigma_z \middle| \vec{k},\sigma,\tau \right\rangle$$

The band dispersion depends on the total gap $\tilde{\Delta} = \Delta + \Delta_H$

Twisted bilayer graphene

Hartree potential

$$\mathrm{V}_H(\vec{r}) = \sum_{\vec{G}} \frac{2\pi e^2}{\epsilon \left|\vec{G}\right|} \rho_{\vec{G}}\, e^{i\vec{G}\vec{r}}$$

$$\rho_{\vec{G}} = \sum_{\vec{k},\sigma,\tau \in occ} \left\langle \vec{k},\sigma,\tau \middle| e^{i\vec{G}\vec{r}} \middle| \vec{k},\sigma,\tau \right\rangle$$

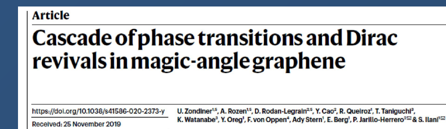
The band dispersion depends on $\left\langle \vec{k},\sigma,\tau \middle| e^{i\vec{G}\vec{r}} \middle| \vec{k},\sigma,\tau \right\rangle$

Exchange term: broken symmetry phases


PHYSICAL REVIEW B **102**, 045107 (2020)

Band structure and insulating states driven by Coulomb interaction in twisted bilayer graphene

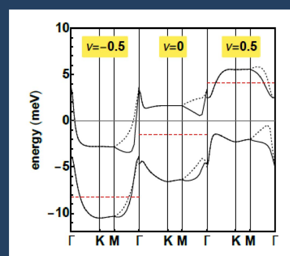
Tommaso Cea[1] and Francisco Guinea[1,2]


Article

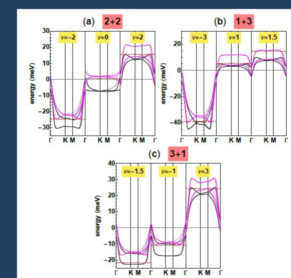
Cascade of phase transitions and Dirac revivals in magic-angle graphene

https://doi.org/10.1038/s41586-020-2373-y
Received: 25 November 2019

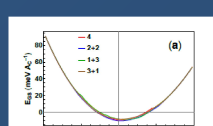
Nature **582**, 203 (2020).

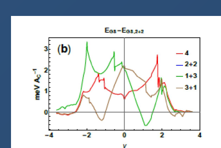


- Gapped phase at half filling
- Breaks C_2 symmetry

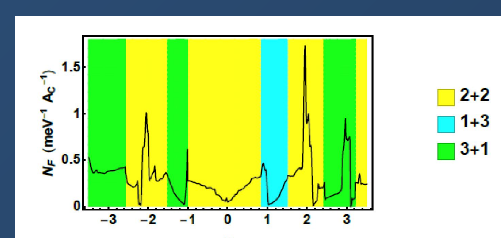


- Spin and/or valley polarized phases near integer fillings
- Metallic. Small Fermi pockets





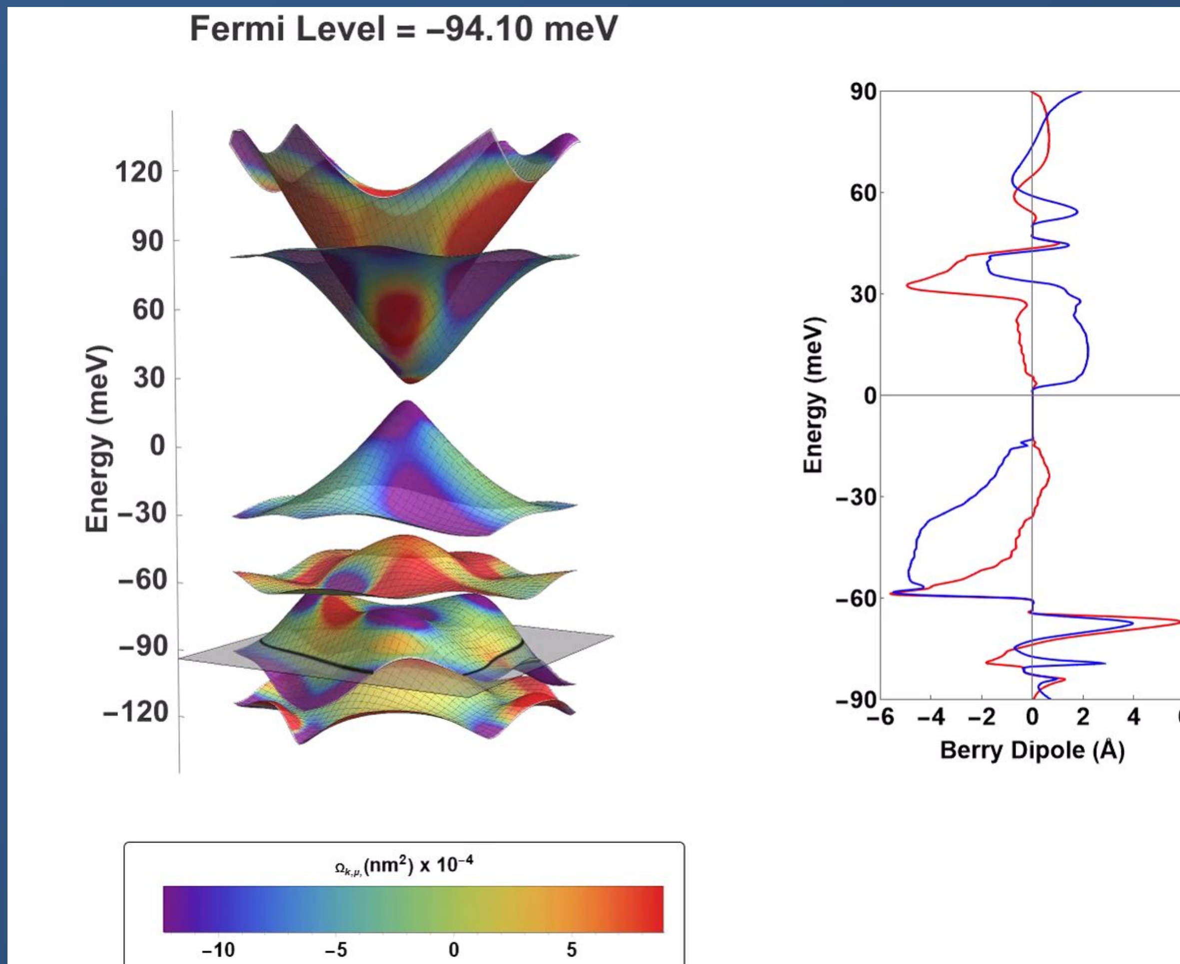
- Small energy differences



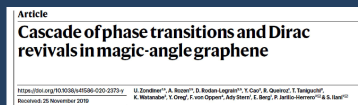
Interaction-Enhanced Topological Hall Effects in Strained Twisted Bilayer Graphene

Pierre A. Pantaleón,[1,*] Võ Tiến Phong,[2,†] Gerardo G. Naumis,[3] and Francisco Guinea[4,5]

arXiv:2204.09619

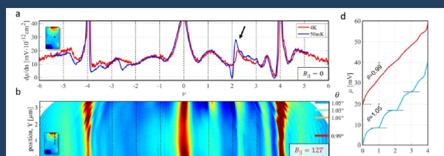


Cascade of phase transitions in twisted bilayer graphene


Article

Cascade of phase transitions and Dirac revivals in magic-angle graphene

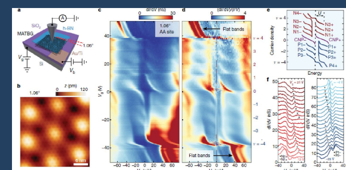
Nature **582**, 203 (2020).

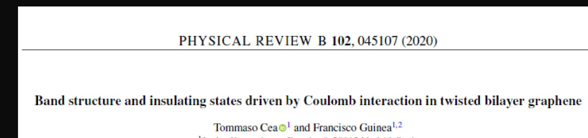



Article

Cascade of electronic transitions in magic-angle twisted bilayer graphene

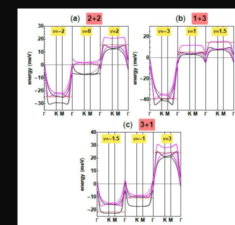
Nature **582**, 198 (2020)



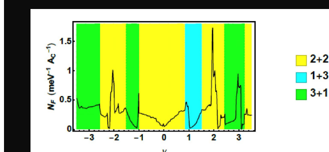

PHYSICAL REVIEW B **102**, 045107 (2020)

Band structure and insulating states driven by Coulomb interaction in twisted bilayer graphene

Tommaso Cea[1] and Francisco Guinea[1,2]

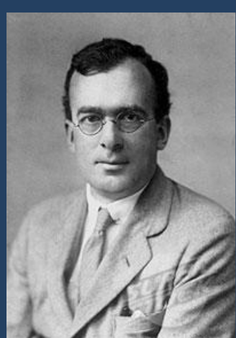


The mean field (HF) cascade

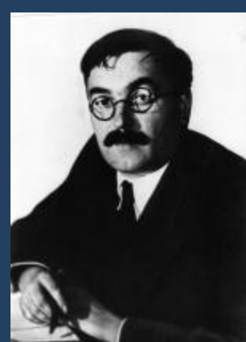



nature materials

ARTICLES



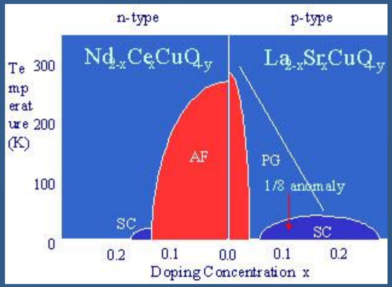
Douglas R. Hartree



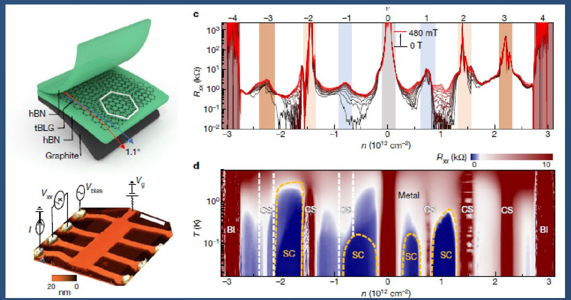
Vladimir A. Fock

- The long range Coulomb interaction gives a qualitative explanation of the band deformations, and of the broken symmetry phases of twisted bilayer graphene.
- It does not identify details of the broken symmetry phases.

Superconductivity and repulsive interactions



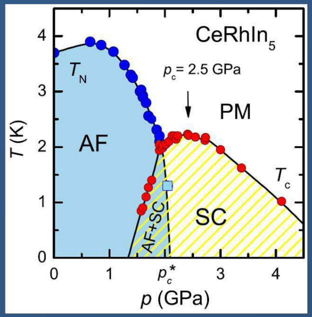
Cuprate superconductors



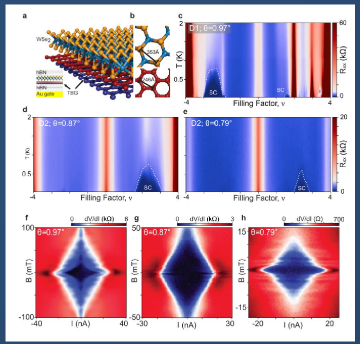
Nature **574**, 653 (2019)

Pairing due to magnetic fluctuations
Variations of the Hubbard model

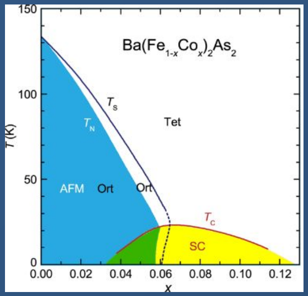
?



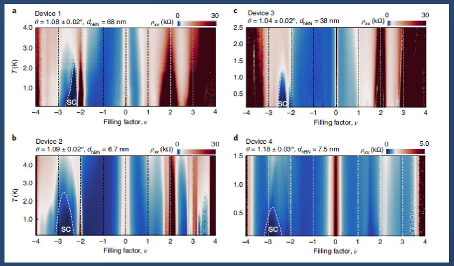
Heavy fermion compounds



Nature **583**, 579 (2020)



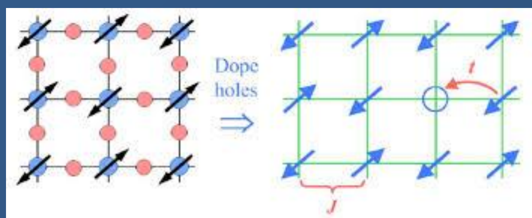
Pnictides



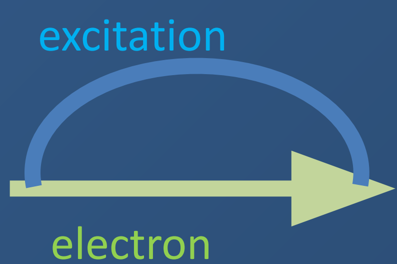
Nature Phys. **16**, 926 (2020)

Superconductivity and excitations from a broken symmetry state

Quasiparticles dressed by excitations



Hubbard ⟶ t-J model
Cuprate oxides



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- Y.-Z. You and A. Vishwanath, ***Superconductivity from valley fluctuations and approximate so(4) symmetry in a weak coupling theory of twisted bilayer graphene***, npj Quantum Materials **4**, 16 (2019).
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- E. Khalaf, N. Bultinck, A. Vishwanath, and M. P. Zaletel, ***Soft modes in magic angle twisted bilayer graphene***, arXiv:2009.14827 (2020),
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- H. Dai, J. Hou, X. Zhang, Y. Liang, and T. Ma, ***Mott insulating state and d + id superconductivity in an abc graphene trilayer***, Phys. Rev. B 104, 035104 (2021).
- S. Chatterjee, T.Wang, E. Berg, and M. P. Zaletel, ***Intervalley coherent order and isospin fluctuation mediated superconductivity in rhombohedral trilayer graphene***, arXiv:2109.00002 (2021).
- Z. Dong, L. Levitov, ***Superconductivity in the vicinity of a spin polarized state in a cubic Dirac band***, arXiv:2109.01133 (2021).

Long range interactions and superconductivity: the Kohn-Luttinger mechanism

Some results which describe pairing channels using perturbative/diagrammatic analyses of the Coulomb interaction

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- Y. Sherkunov and J. J. Betouras, ***Electronic phases in twisted bilayer graphene at magic angles as a result of van hove singularities and interactions***, Phys. Rev. B **98**, 205151 (2018).
- J. González and T. Stauber, ***Kohn-Luttinger superconductivity in twisted bilayer graphene***, Phys. Rev. Lett. **122**, 026801 (2019).
- B. Roy and V. Juricic, ***Unconventional superconductivity in nearly flat bands in twisted bilayer graphene***, Physical Review B **99**, 12 1407 (2019).
- D. V. Chichinadze, L. Classen, and A. V. Chubukov, ***Nematic superconductivity in twisted bilayer graphene***, Phys. Rev. B **101**, 224513 (2020).
- Y.-P. Lin and R. M. Nandkishore, ***Parquet renormalization group analysis of weak-coupling instabilities with multiple high-order van hove points inside the Brillouin zone***, Phys. Rev. B **102**, 245122 (2020).
- C. Lewandowski, D. Chowdhury, and J. Ruhman, ***Pairing in magic-angle twisted bilayer graphene: role of phonon and plasmon umklapp***, (2020), arXiv:2007.15002.
- W. Qin, B. Zou, and A. H. MacDonald, ***Critical magnetic fields and electron-pairing in magic-angle twisted bilayer graphene***, (2021), arXiv:2102.10504.
- C. Lewandowski, S. Nadj-Perge, and D. Chowdhury, ***Does filling-dependent band renormalization aid pairing in twisted bilayer graphene?***, (2021), arXiv:2102.05661.

to long-range effects.

Superconductivity from repulsive interactions: interband transition

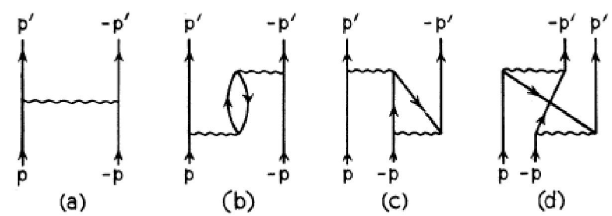
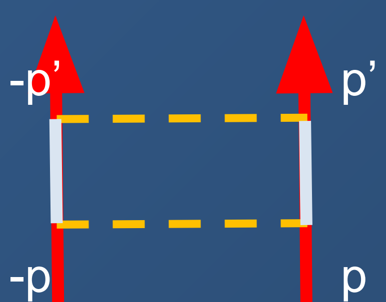


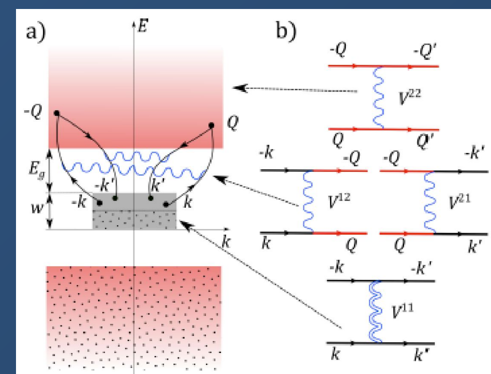
FIG. 1. Types of particle-particle interaction diagrams up to the second order which contribute to the irreducible scattering vertex.



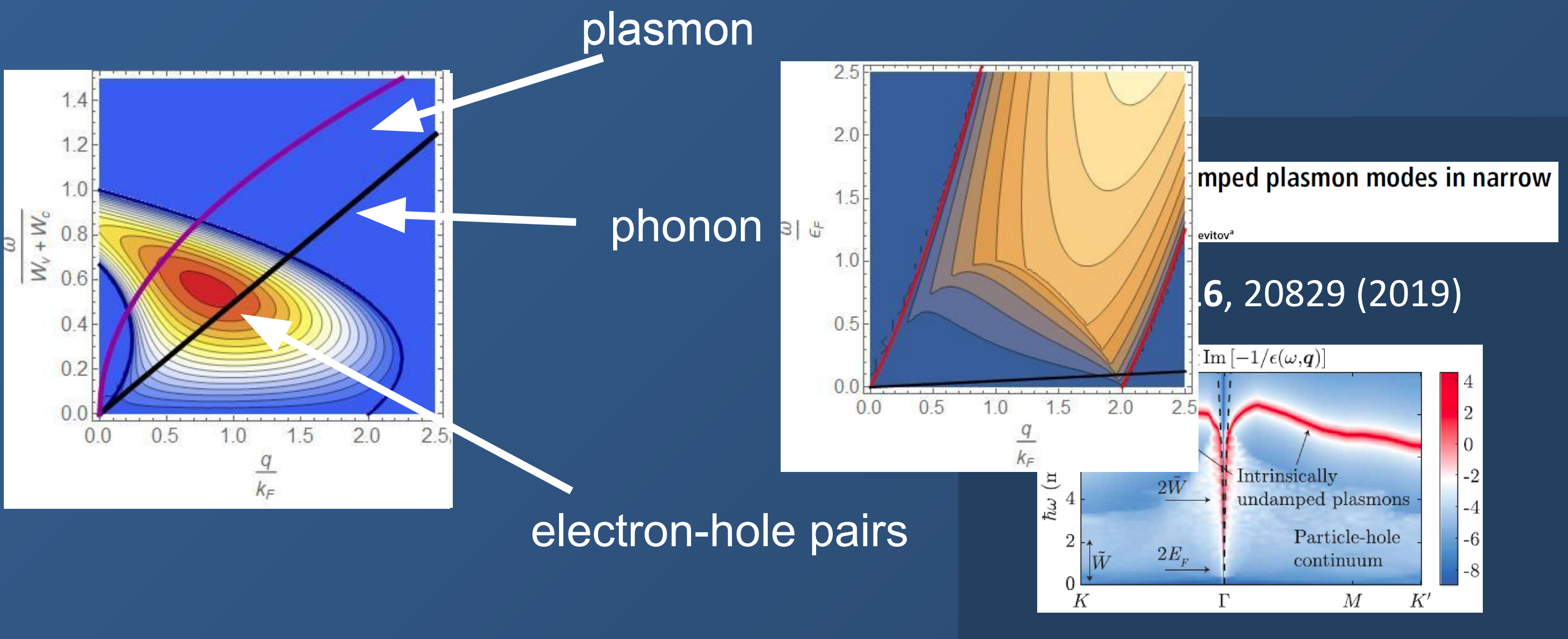
Activating superconductivity in a repulsive system by high-energy degrees of freedom

Zhiyu Dong and Leonid Levitov

arXiv:2103.08767



Low energy excitations and charge excitations: Electron-hole pairs, plasmons, and phonons



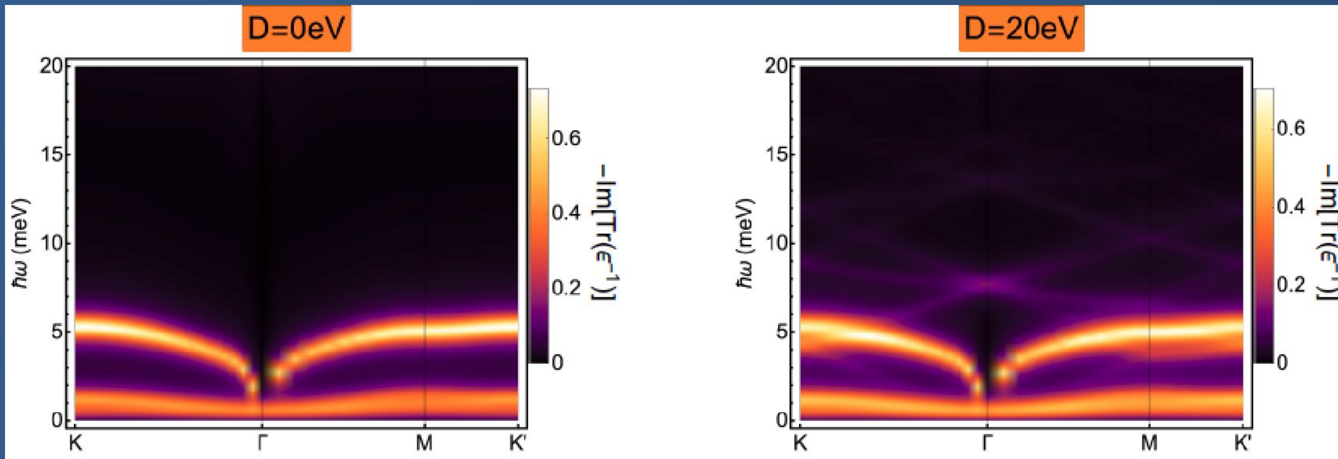
- Longitudinal phonon with displacements of the same sign in the two layers. Coupling through the deformation potential, *D*=20 eV.
- Longitudinal phonons with displacements of opposite sign do not induce global charge modulations.
- Transverse phonons couple to the velocity operator.

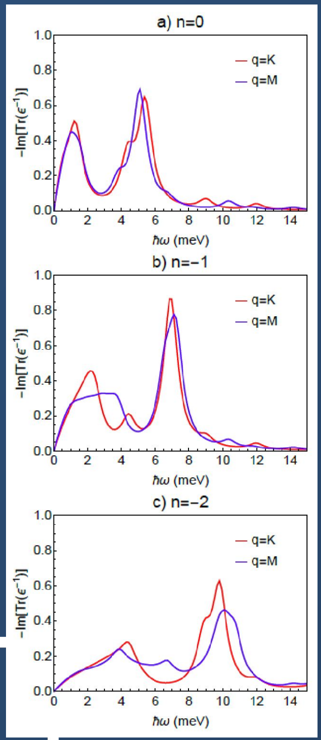
Longitudinal phonons and electron-hole pairs

Coulomb interaction, phonons, and superconductivity in twisted bilayer graphene

Tommaso Cea[a,b] and Francisco Guinea[a,c,d,1]

PNAS 2021 Vol. 118 No. 32 e2107874118





$$1-\left(\frac{2\pi e^2}{\epsilon|\vec{q}|}+\frac{D^2}{\lambda+2\mu}\frac{\omega_{\vec{q}}^2}{\omega^2-\omega_{\vec{q}}^2}\right)\chi_0(\vec{q},\omega)=0$$

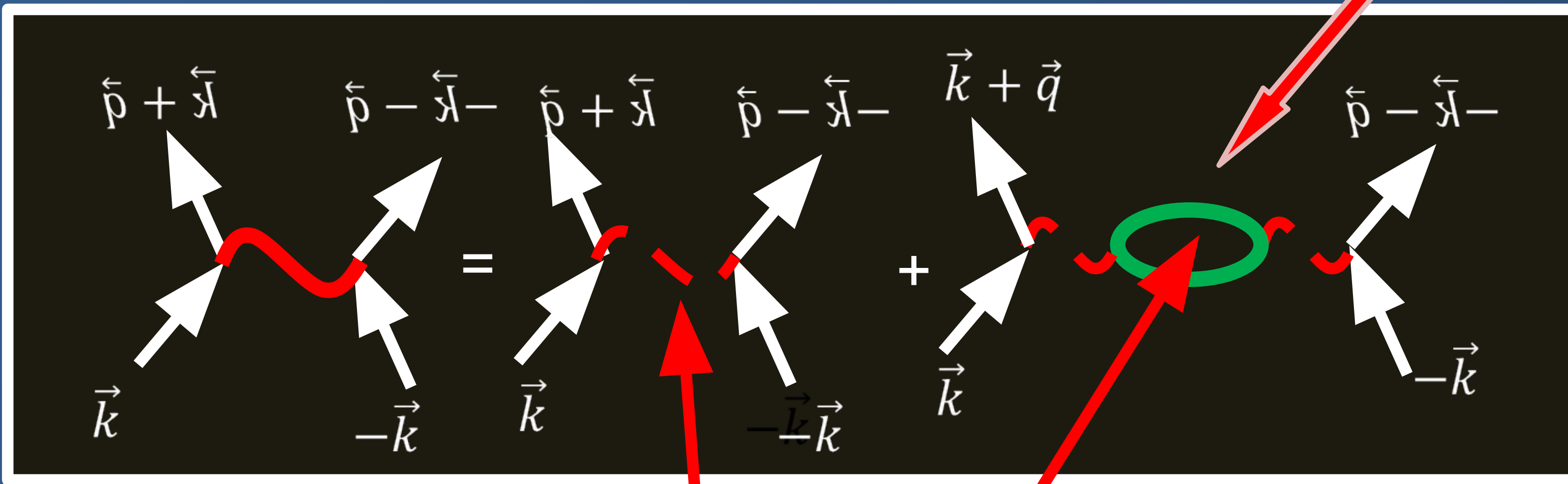
$$\tilde{\omega}_{\vec{q}}^2\approx\omega_{\vec{q}}^2+\omega_{\vec{q}}^2\frac{\frac{D^2}{\lambda+2\mu}\chi_0(\vec{q},\omega)}{1-\frac{2\pi e^2}{\epsilon|\vec{q}|}\chi_0(\vec{q},\omega)}\approx\omega_{\vec{q}}^2\left[1-\frac{\frac{D^2}{\lambda+2\mu}\mathcal{D}(\epsilon_F)}{1+\frac{2\pi e^2}{\epsilon|\vec{q}|}\mathcal{D}(\epsilon_F)}\right]\approx\omega_{\vec{q}}^2\left[1-\frac{\frac{D^2}{\lambda+2\mu}\frac{\mathcal{N}}{WL^2}}{1+\frac{2\pi e^2}{\epsilon|\vec{q}|}\frac{\mathcal{N}}{WL^2}}\right]$$

$$\tilde{g}\approx\frac{D^2}{\lambda+2\mu}\frac{\mathcal{N}}{WL^2}\approx 0.4$$

$$D=20\text{ eV}$$
$$\lambda+2\mu\approx 20\text{ eV}\text{Å}^{-2}$$
$$\mathcal{N}=4$$
$$W\approx 10\text{ meV}$$
$$L^2\approx(140\text{ Å})^2$$

Repulsive interactions and pairing

RPA resummation



- Longitudinal phonons modify the susceptibility.
- The bare interaction is the repulsive Coulomb coupling

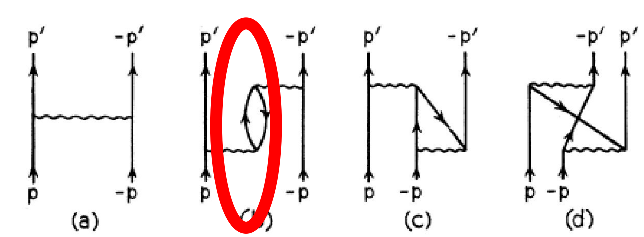
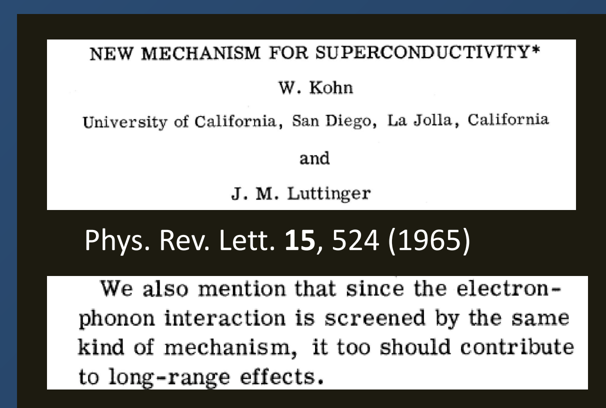


FIG. 1. Types of particle-particle interaction diagrams up to the second order which contribute to the irreducible scattering vertex.



NEW MECHANISM FOR SUPERCONDUCTIVITY*

W. Kohn
University of California, San Diego, La Jolla, California
and
J. M. Luttinger

Phys. Rev. Lett. **15**, 524 (1965)

We also mention that since the electron-phonon interaction is screened by the same kind of mechanism, it too should contribute to long-range effects.

Gap equation

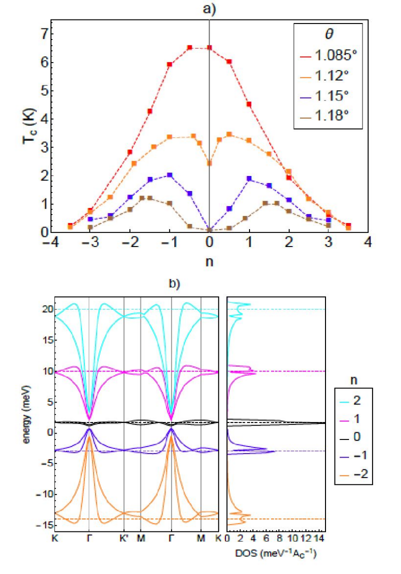
$$\widetilde{\Delta}^{m_1,m_2}_{\alpha,\beta}\left(\vec{k}\right) = \sum_{n_1,n_2,\vec{q}} \Gamma^{m_1,m_2}_{n_1,n_2,\alpha,\beta}\left(\vec{k},\vec{k}+\vec{q}\right)\widetilde{\Delta}^{n_1,n_2}_{\alpha,\beta}\left(\vec{k}+\vec{q}\right)$$

$$\mathcal{M}_{\vec{G}}\left(\vec{k},\vec{k}+\vec{q}\right) = \int d^2\vec{r}\, u^*_{\vec{k}}(\vec{r})e^{i\vec{G}\vec{r}}u_{\vec{k}+\vec{q}}(\vec{r})$$

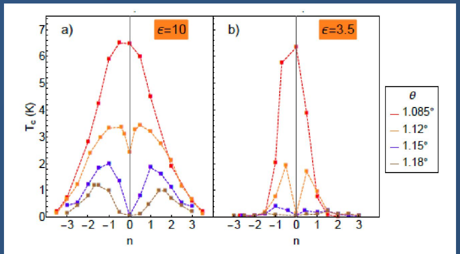
$$\Gamma^{m_1,m_2}_{n_1,n_2,\alpha,\beta}\left(\vec{k},\vec{k}+\vec{q}\right) = -\frac{1}{\Omega}\sum_{\vec{G}_1,\vec{G}_1'}\sum_{\vec{G}_2,\vec{G}_2'}\sum_{i_1,i_2} V^{scr}_{\vec{G}_1-\vec{G}_1',\vec{G}_2-\vec{G}_2'}(\vec{q})\,\mathcal{M}^*_{\vec{G}_1-\vec{G}_1'}\left(\vec{k},\vec{k}+\vec{q}\right)\mathcal{M}_{\vec{G}_2-\vec{G}_2'}\left(\vec{k},\vec{k}+\vec{q}\right)$$
$$\times\sqrt{\frac{f\left(-E_{m_2,-\vec{k},\beta}+\mu\right)-f\left(E_{m_1,\vec{k},\alpha}-\mu\right)}{E_{m_2,-\vec{k},\beta}+E_{m_1,\vec{k},\alpha}-2\mu}}\sqrt{\frac{f\left(-E_{n_2,-\vec{k}-\vec{q},\beta}+\mu\right)-f\left(E_{n_1,\vec{k}+\vec{q},\alpha}-\mu\right)}{E_{m_2,-\vec{k}-\vec{q},\beta}+E_{m_1,\vec{k}+\vec{q},\alpha}-2\mu}}$$

Results: critical temperature

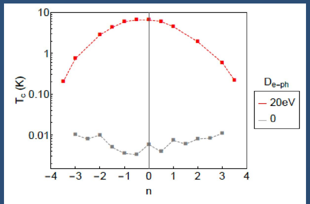
Critical temperatures



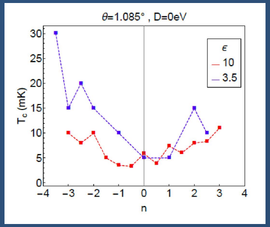
Bands, densities of states, magic angle



Dependence on dielectric constant



Critical temperature with and without e-ph interaction



Critical temperature for e-e interaction only

Superconductivity in a doped valley coherent insulator in magic angle graphene: Goldstone-mediated pairing and Kohn-Luttinger mechanism

Vladyslav Kozii,[1,2] Michael P. Zaletel,[1,2] and Nick Bultinck[1,3]

arXiv:2005.12961

Using $\varepsilon_F = 3.2$ meV and $\lambda \approx 0.08$, one finds $T_c \approx 1.3 \times 10^{-4}$ K, which is too low compared to the experimental values of $T_c \approx 0.3$ K. However, because of the

- The critical temperature is significantly enhanced by the electron-phonon interaction.
- Superconductivity correlates with the density of states at the Fermi level.
- The effect of external screening depends on the strength of the electron-phonon interaction.

Nature of the pairing interaction

Electronic susceptibility:

$$\chi_{\vec{G},\vec{G}'}(\vec{q},\omega) = \sum_{\alpha,\beta,\vec{k}} \left[\mathcal{M}_{\vec{G}}^{\alpha,\beta}\left(\vec{k},\vec{k}+\vec{q}\right)\right]^{*} \mathcal{M}_{\vec{G}'}^{\alpha,\beta}\left(\vec{k},\vec{k}+\vec{q}\right) \frac{n^{\beta}_{\vec{k}+\vec{q}} - n^{\alpha}_{\vec{k}}}{\omega - \epsilon^{\beta}_{\vec{k}+\vec{q}} + \epsilon^{\alpha}_{\vec{k}}}$$

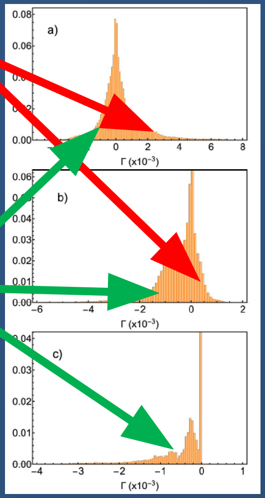
Form factor: $\mathcal{M}_{\vec{G}}^{\alpha,\beta}\left(\vec{k},\vec{k}+\vec{q}\right) = \left\langle \vec{k},\alpha \middle| e^{i\vec{G}\vec{r}} \middle| \vec{k}+\vec{q},\beta \right\rangle$

Superconducting kernel: $\Gamma_{\alpha,\beta}\left(\vec{k},\vec{k}+\vec{q}\right)$

- Umklapp processes are crucial.
- Form factors lead to attractive interactions.
- The order parameter does not change sign.
- Consistent with spin singlet/valley triplet or spin triplet/valley singlet superconductivity.

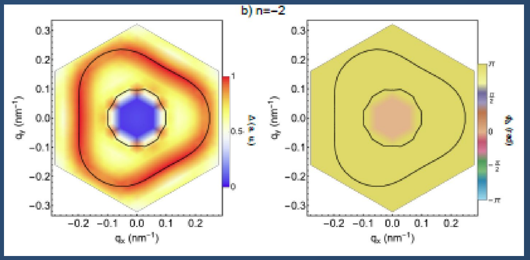
Attraction

Repulsion



Distribution of the matrix elements of the superconducting kernel

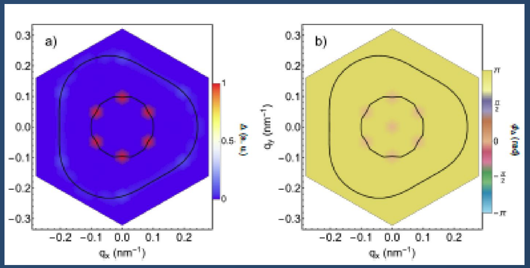
Including phonons



Excluding phonons

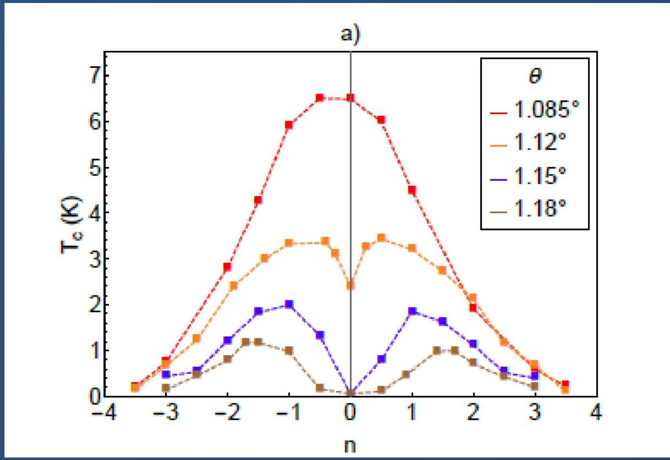
Excluding phonons and simplifying the form factors

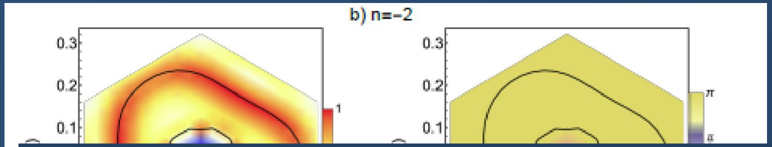
$$\mathcal{M}_{\vec{G}}^{\alpha,\beta}\left(\vec{k},\vec{k}+\vec{q}\right) \approx \delta_{\alpha,\beta}\, g\left(\vec{k},\vec{k}+\vec{q}\right) f_{\vec{G}}$$



Other effects

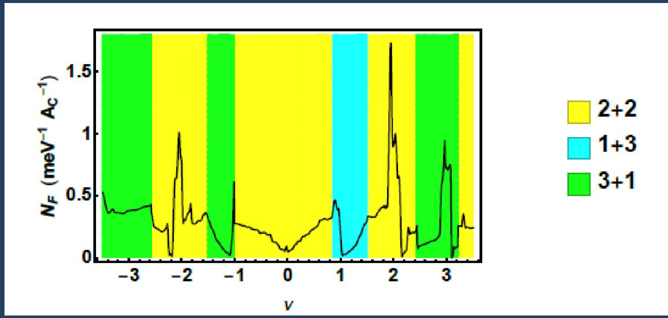
- Transverse acoustical and optical phonons are not included.
  Possible enhancement of T_c.
- No exchange effects. Spin and/or valley polarized phases not considered.
  Calculation approximately correct for spin polarized phases with equal occupancy of the K and K' valleys, such as the 2+2 phase near ν=2.
  No soft spin and/or valley modes.
- No retardation effects.
  Upper bound on the critical temperature, $k_B T_c \leq \hbar\omega_{ph}$.





Superconducting properties

Mechanism intrinsic to twisted bilayer graphene.
Multigap superconductor.
No sign changes in the order parameter within each valley.
Weak pair breaking due to elastic scattering.

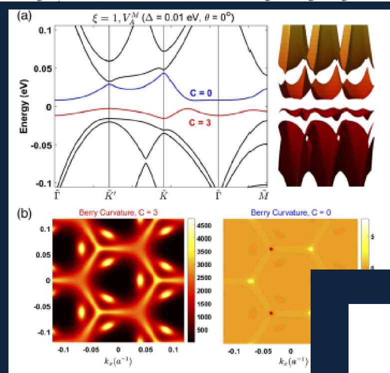


Flat bands and interactions in a graphene trilayer on a substrate

PHYSICAL REVIEW LETTERS 122, 016401 (2019)

Gate-Tunable Topological Flat Bands in Trilayer Graphene Boron-Nitride Moiré Superlattices

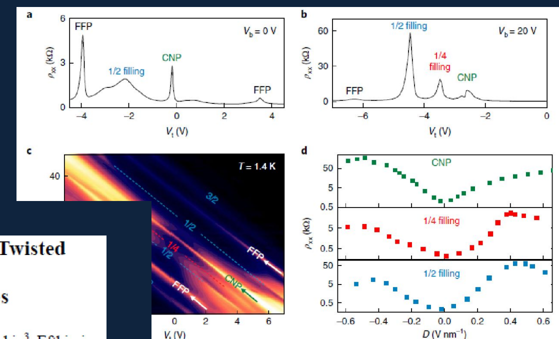
Bheema Lingam Chittari,[1,2] Guorui Chen,[2] Yuanbo Zhang,[3,4] Feng Wang,[2] and Jeil Jung[1,2]



Evidence of a gate-tunable Mott insulator in a trilayer graphene moiré superlattice

Guorui Chen[1,2,3], Lili Jiang[1], Shuang Wu[4], Bosai Lyu[3,5], Hongyuan Li[3,5], Bheema Lingam Chittari[6], Kenji Watanabe[7], Takashi Taniguchi[7], Zhiwen Shi[3,5], Jeil Jung[6], Yuanbo Zhang[2,3,8]* and Feng Wang[1,9,10]*

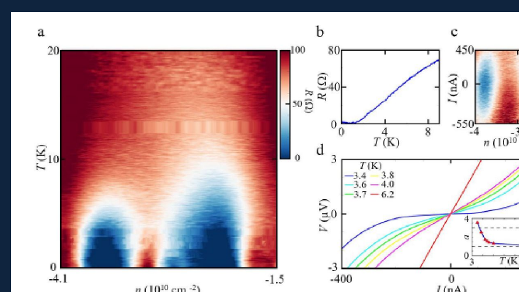
Nature Phys. **15**, 237 (2019)



Correlated Superconducting and Insulating States in Twisted Trilayer Graphene Moiré of Moiré Superlattices

Kan-Ting Tsai[1†], Xi Zhang[1†], Ziyan Zhu[2], Yujie Luo[1], Stephen Carr[2], Mitchell Luskin[3], Efthimios Kaxiras[2,4], Ke Wang[1]*

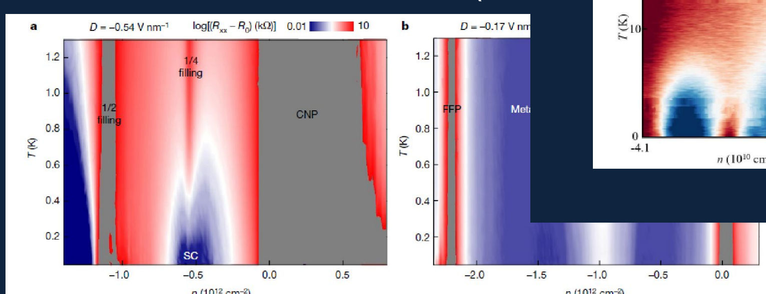
arXiv:1912.03375



Signatures of tunable superconductiv[ity in trilayer] graphene moiré superlattice

Guorui Chen[1,2,13], Aaron L. Sharpe[3,4,13], Patrick Gallagher[1,2], Ilan T. Rosen[3,4], Eli J. Fox[4,5], Lili[…] Hongyuan Li[6,7], Kenji Watanabe[8], Takashi Taniguchi[8], Jeil Jung[9], Zhiwen Shi[6,7], David Goldh[…] Yuanbo Zhang[7,10,11]* & Feng Wang[1,2,12]*

Nature **572**, 215 (20[19])



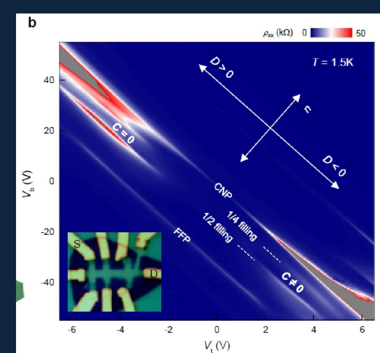
Article

Tunable correlated Chern insulator and ferromagnetism in a moiré superlattice

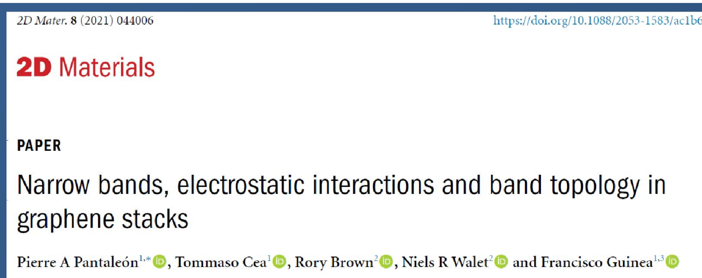
https://doi.org/10.1038/s41586-020-2049-7
Received: 16 May 2019
Accepted: 11 December 2019

Guorui Chen[1,2], Aaron L. Sharpe[3,4], Eli J. Fox[4,5], Ya-Hui Zhang[6], Shaoxin Wang[7], Lili Jiang[2], Bosai Lyu[7,8], Hongyuan Li[7,8], Kenji Watanabe[9], Takashi Taniguchi[9], Zhiwen Shi[7,8], T. Senthil[6], David Goldhaber-Gordon[4,5], Yuanbo Zhang[8,10,11], & Feng Wang[1,2,12]

Nature **579**, 56 (2020)



Flat bands and interactions in an ABC trilayer on hBN

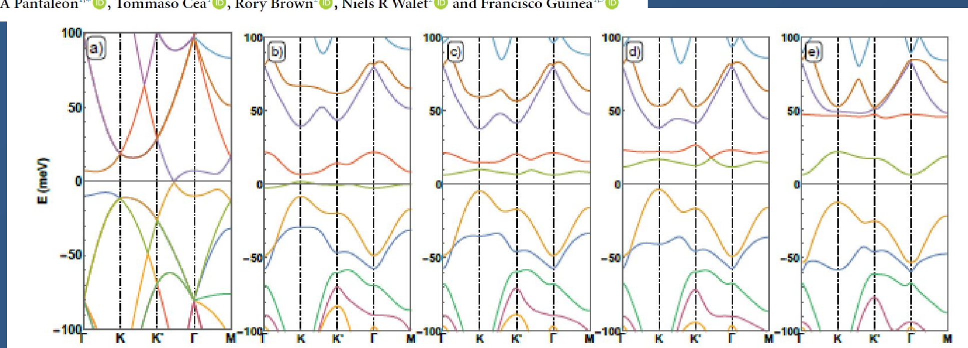

2D Mater. 8 (2021) 044006 https://doi.org/10.1088/2053-1583/ac1b6c

2D Materials

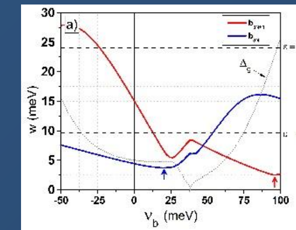
PAPER

Narrow bands, electrostatic interactions and band topology in graphene stacks

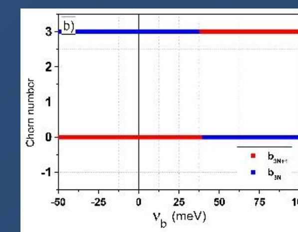
Pierre A Pantaleón[1,*], Tommaso Cea[1], Rory Brown[2], Niels R Walet[2] and Francisco Guinea[1,3]



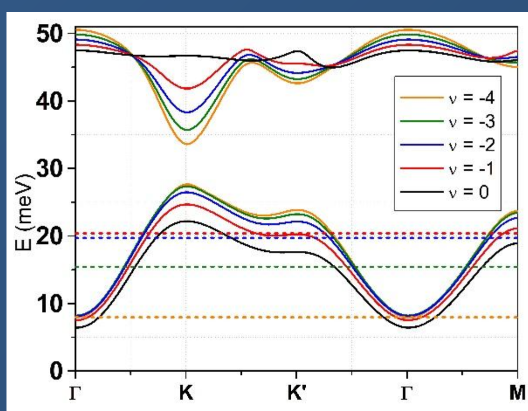
Non interacting bands as function of electrostatic bias



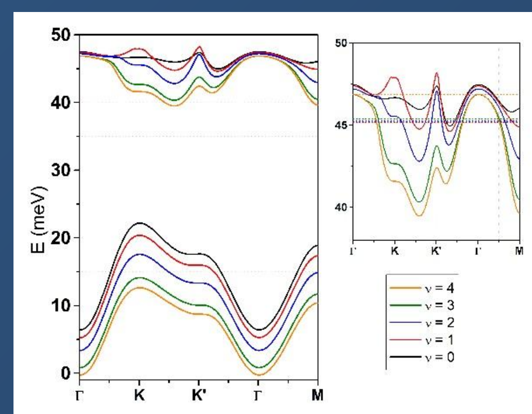
Bandwidth of the two central bands



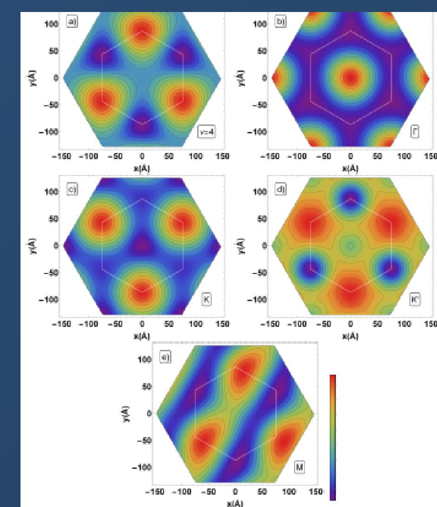
Chern number of the two central bands



Lower band



Upper band



Charge densities

Band dispersion as function of filling. Self consistent results.

Flat bands and interactions in two graphene bilayers with a twist

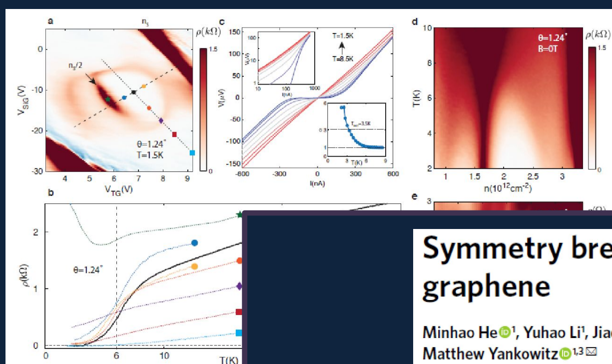
Article

Tunable spin-polarized correlated states in twisted double bilayer graphene

https://doi.org/10.1038/s41586-020-2458-7
Received: 25 March 2019

Xiaomeng Liu[1,3]✉, Zeyu Hao[1], Eslam Khalaf[1], Jong Yeon Lee[1], Yuval Ronen[1], Hyobin Yoo[1], Danial Haei Najafabadi[1], Kenji Watanabe[2], Takashi Taniguchi[2], Ashvin Vishwanath[1] & Philip Kim[1]✉

Nature **583**, 221 (2020)

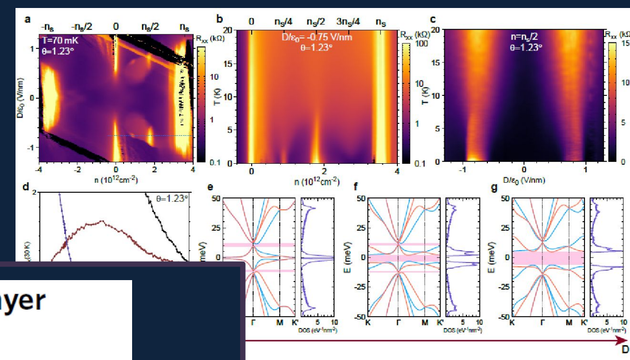


Tunable correlated states and spin-polarized phases in twisted bilayer–bilayer graphene

https://doi.org/10.1038/s41586-020-2260-6
Received: 24 March 2019

Yuan Cao[1]✉, Daniel Rodan-Legrain[1], Oriol Rubies-Bigorda[1], Jeong Min Park[1], Kenji Watanabe[2], Takashi Taniguchi[2] & Pablo Jarillo-Herrero[1]✉

Nature **583**, 215 (2020)



Symmetry breaking in twisted double bilayer graphene

Minhao He[1], Yuhao Li[1], Jiaqi Cai[1], Yang Liu[1], K. Watanabe[2], T. Taniguchi[2], Xiaodong Xu[1,3]✉ and Matthew Yankowitz[1,3]✉

Nature Phys. (2020) 10.1038/s41567-020-1030-6

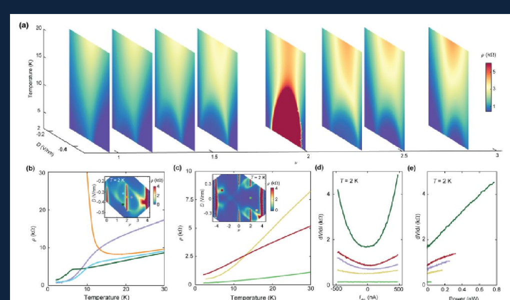
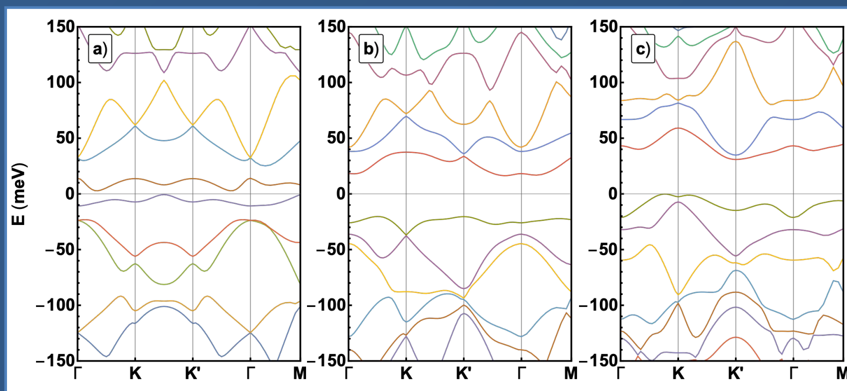


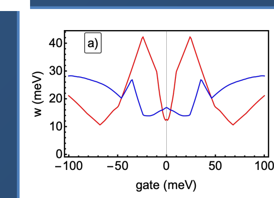
FIG. 2. **Temperature-dependent transport and Joule heating in tDBG. a**, Resistivity as a function of temperature

Flat bands and interactions in a AB-AB twisted double bilayer graphene

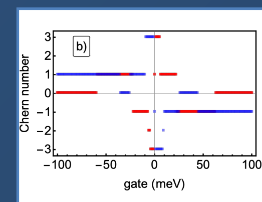

2D Mater. 8 (2021) 044006
https://doi.org/10.1088/2053-1583/ac1b6c
2D Materials
PAPER
Narrow bands, electrostatic interactions and band topology in graphene stacks
Pierre A Pantaleón[1,*], Tommaso Cea[1], Rory Brown[2], Niels R Walet[2] and Francisco Guinea[1,3]



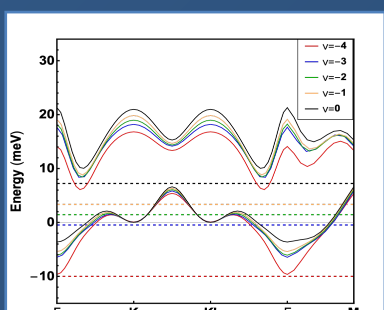
Non interacting bands as function of electrostatic bias



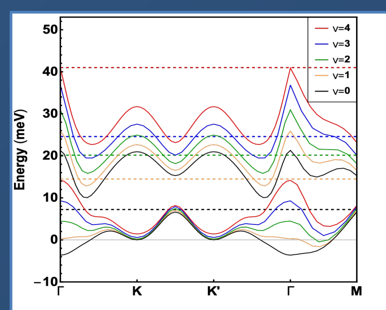
Bandwidth of the two central bands



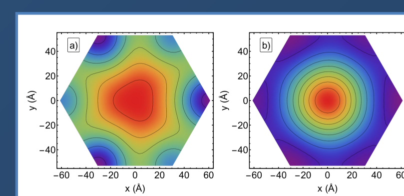
Chern number of the two central bands



Lower band



Upper band



Charge densities

Band dispersion as function of filling. Self consistent results.

Superconductivity in twisted trilayers

Article

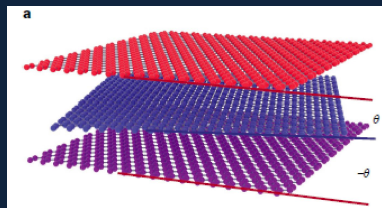
Tunable strongly coupled superconductivity in magic-angle twisted trilayer graphene

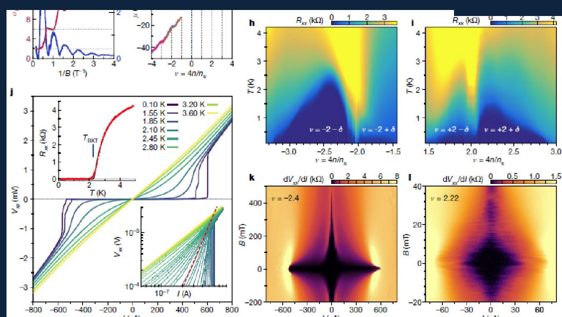
https://doi.org/10.1038/s41586-021-03192-0

Jeong Min Park[1,4], Yuan Cao[1,4], Kenji Watanabe[2], Takashi Tanig
Pablo Jarillo-Herrero[1]

Received: 26 October 2020

Nature | Vol 590 | 11 February 2021



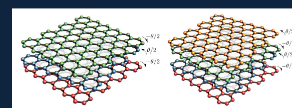


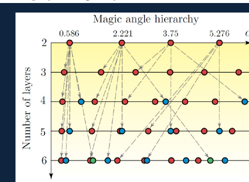
PHYSICAL REVIEW B **100**, 085109 (2019)

Editors' Suggestion

Magic angle hierarchy in twisted graphene multilayers

Eslam Khalaf,* Alex J. Kruchkov, Grigory Tarnopolsky, and Ashvin Vishwanath



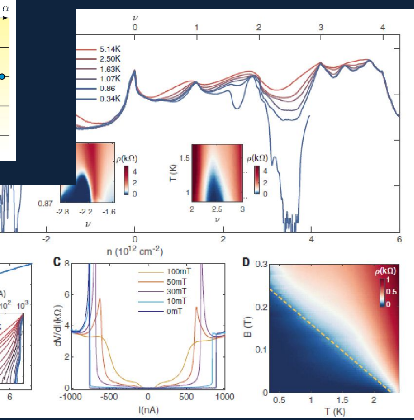


SUPERCONDUCTIVITY

Electric field–tunable superconductivity in alternating-twist magic-angle trilayer graphene

...ick Ledwith[1], Eslam Khalaf[1], Danial Haie Najafabadi[1],
Ashvin Vishwanath[1], Philip Kim[1]†

133–1138 (2021) 12 March 2021



Article

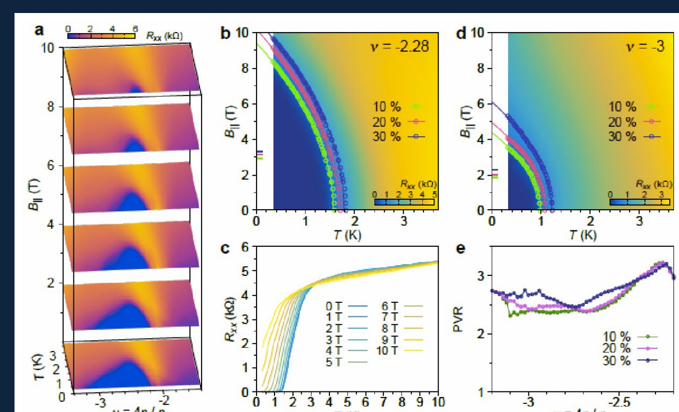
Pauli-limit violation and re-entrant superconductivity in moiré graphene

https://doi.org/10.1038/s41586-021-03685-y

Yuan Cao[1,4], Jeong Min Park[1,4], Kenji Watanabe[2], Takashi Taniguchi[3] & Pablo Jarillo-Herrero[1]

Received: 4 March 2021

528 | Nature | Vol 595 | 22 July 2021

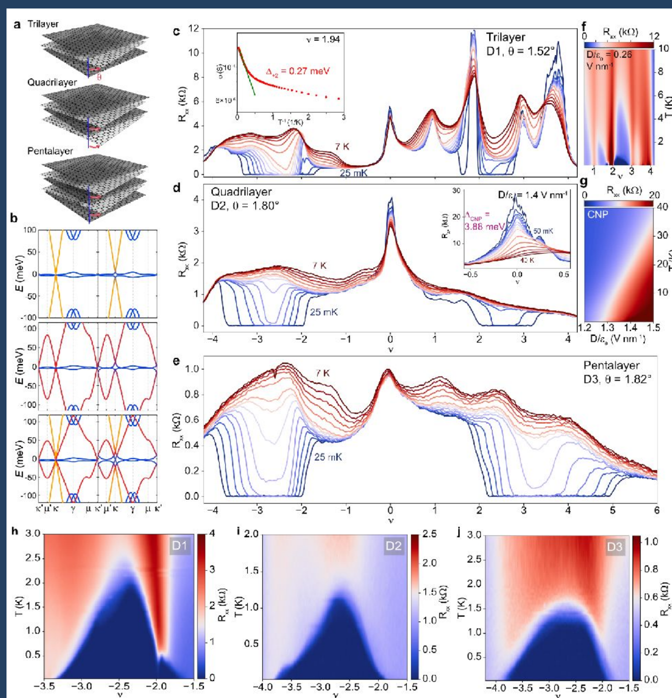


Superconductivity in twisted multilayers

Ascendance of Superconductivity in Magic-Angle Graphene Multilayers

Yiran Zhang[1,2,3*], Robert Polski[1,2*], Cyprian Lewandowski[2,3], Alex Thomson[2,3,4], Yang Peng[5], Youngjoon Choi[1,2,3], Hyunjin Kim[1,2,3], Kenji Watanabe[6], Takashi Taniguchi[6], Jason Alicea[2,3], Felix von Oppen[7], Gil Refael[2,3], and Stevan Nadj-Perge[1,2†]

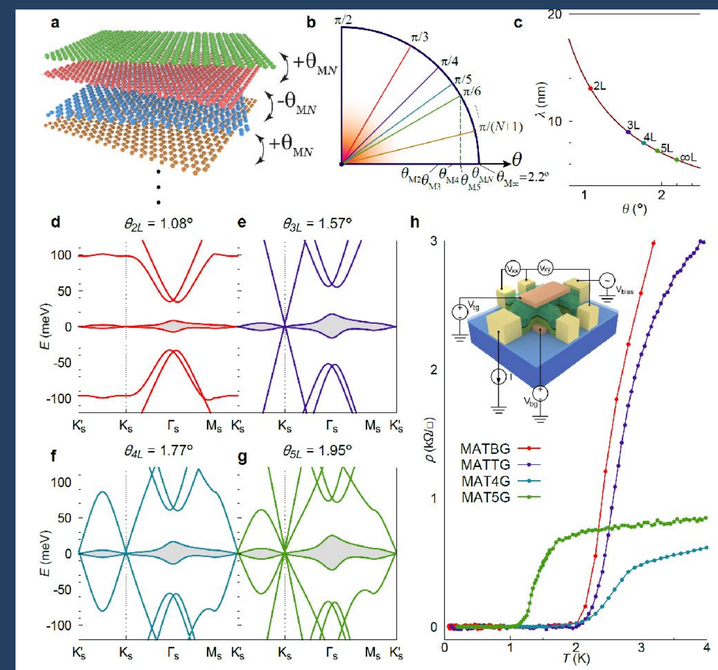
arXiv:2112:09270



Magic-Angle Multilayer Graphene: A Robust Family of Moiré Superconductors

Jeong Min Park,[1,*,†] Yuan Cao,[1,2,*] Liqiao Xia,[1] Shuwen Sun,[1]
Kenji Watanabe,[3] Takashi Taniguchi,[3] and Pablo Jarillo-Herrero[1,†]

arXiv:2112:10760



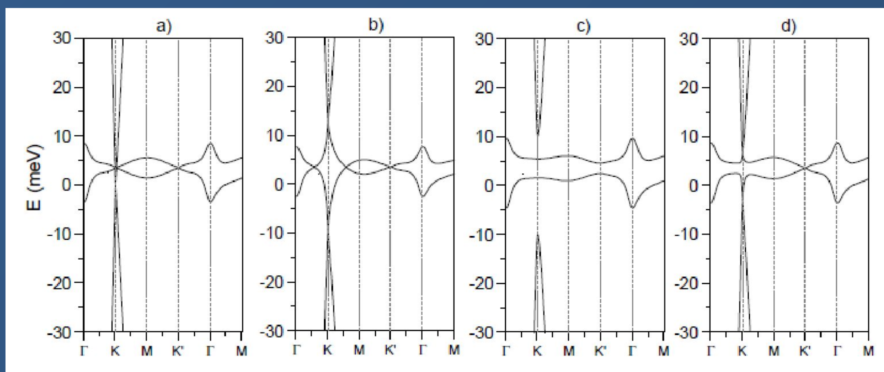
Superconductivity in twisted trilayers: theory

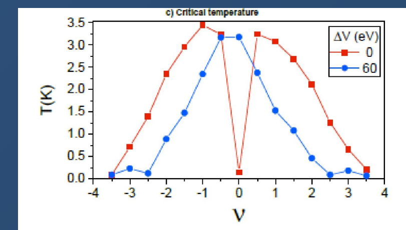
PHYSICAL REVIEW B 104, L121116 (2021)

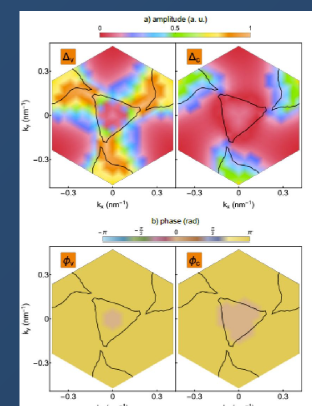
Letter

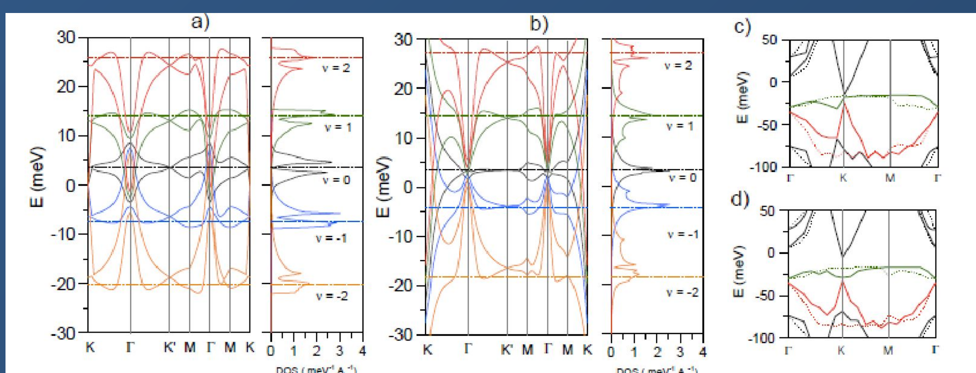
Band structure and superconductivity in twisted trilayer graphene

Võ Tiến Phong,[1] Pierre A. Pantaleón,[2] Tommaso Cea,[2] and Francisco Guinea[2,3]

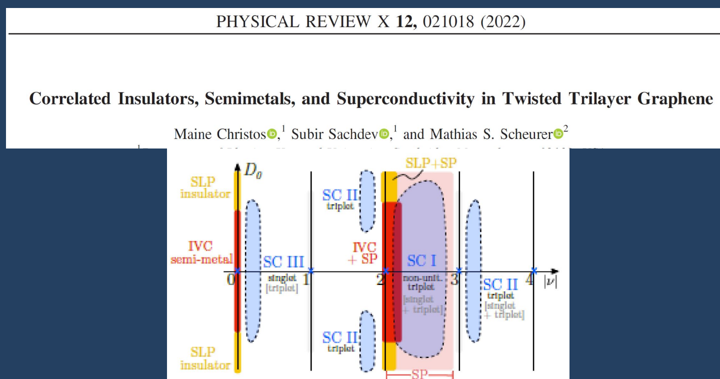






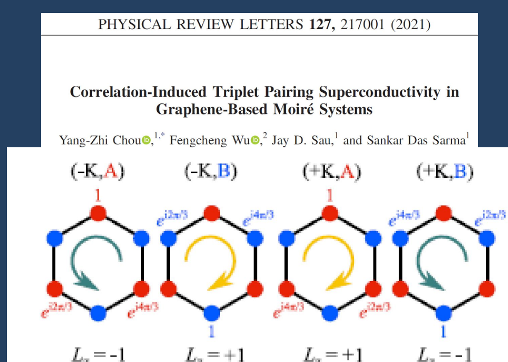


Other theoretical work


PHYSICAL REVIEW X 12, 021018 (2022)

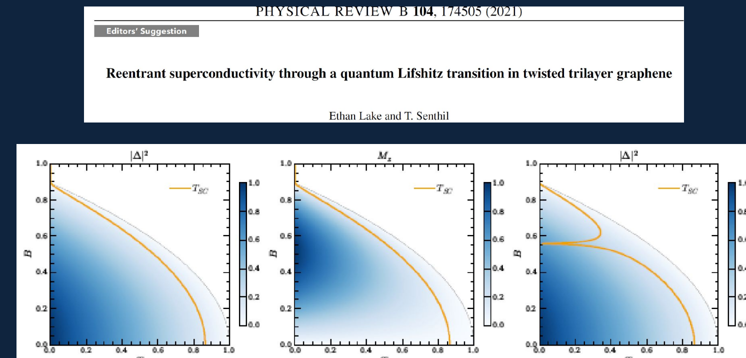
Correlated Insulators, Semimetals, and Superconductivity in Twisted Trilayer Graphene

Maine Christos,[1] Subir Sachdev,[1] and Mathias S. Scheurer[2]


PHYSICAL REVIEW LETTERS 127, 217001 (2021)

Correlation-Induced Triplet Pairing Superconductivity in Graphene-Based Moiré Systems

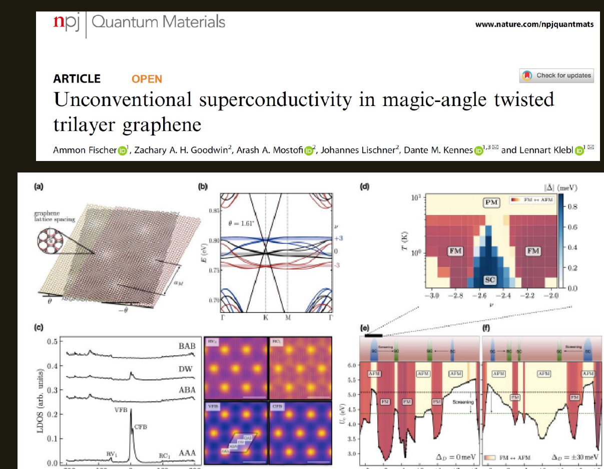
Yang-Zhi Chou,[1,*] Fengcheng Wu,[2] Jay D. Sau,[1] and Sankar Das Sarma[1]


PHYSICAL REVIEW B 104, 174505 (2021)

Editors' Suggestion

Reentrant superconductivity through a quantum Lifshitz transition in twisted trilayer graphene

Ethan Lake and T. Senthil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ARTICLE OPEN

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Unconventional superconductivity in magic-angle twisted trilayer graphene

Ammon Fischer[1], Zachary A. H. Goodwin[2], Arash A. Mostofi[2], Johannes Lischner[2], Dante M. Kennes[1,3] and Lennart Klebl[1]

Superconductivity in graphene without a moiré superlattice

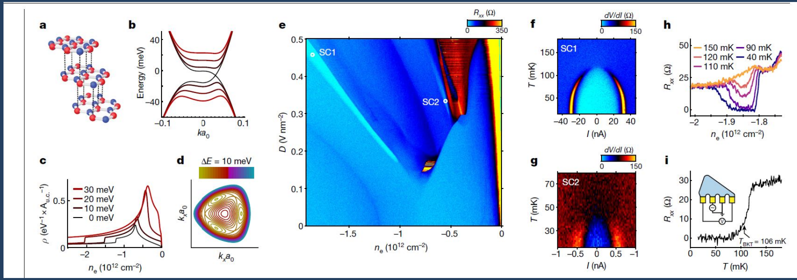
Article

Superconductivity in rhombohedral trilayer graphene

https://doi.org/10.1038/s41586-021-03926-0

Haoxin Zhou[1], Tian Xie[1], Takashi Taniguchi[2], Kenji Watanabe[3] & Andrea F. Young[1]

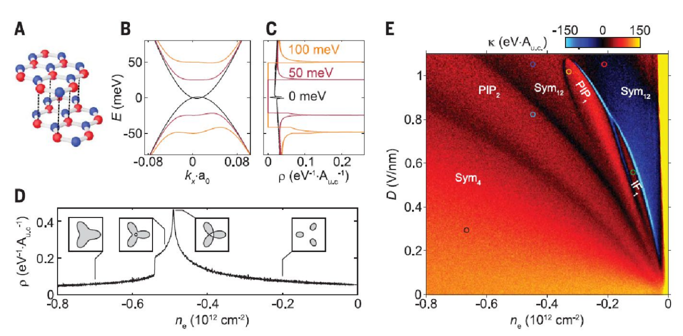
434 | Nature | Vol 598 | 21 October 2021



Isospin magnetism and spin-polarized superconductivity in Bernal bilayer graphene

Haoxin Zhou[1]†, Ludwig Holleis[1], Yu Saito[1], Liam Cohen[1], William Huynh[1], Caitlin L. Patterson[1], Fangyuan Yang[1], Takashi Taniguchi[2], Kenji Watanabe[3], Andrea F. Young[1]*

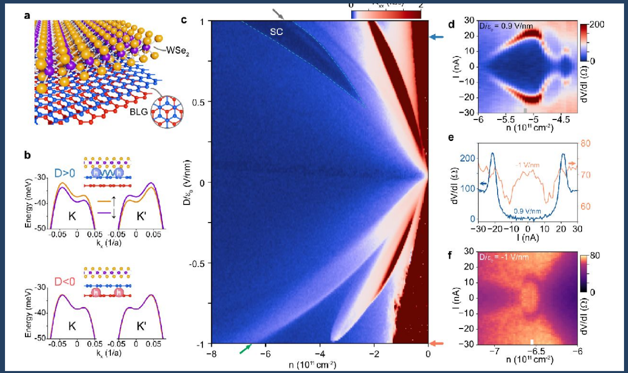
Zhou *et al.*, *Science* **375**, 774–778 (2022) 18 February 2022



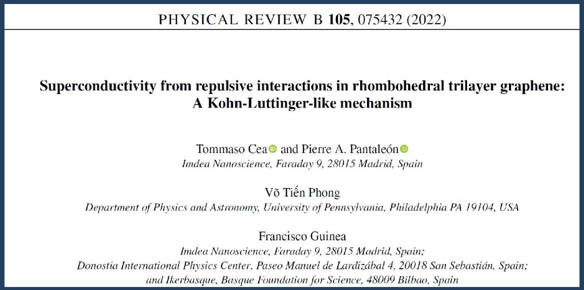
Spin-Orbit Enhanced Superconductivity in Bernal Bilayer Graphene

Yiran Zhang[1,2,3], Robert Polski[1,2], Alex Thomson[2,3,4], Étienne Lantagne-Hurtubise[2,3], Cyprian Lewandowski[2,3], Haoxin Zhou[1,2], Kenji Watanabe[5], Takashi Taniguchi[5], Jason Alicea[2,3], and Stevan Nadj-Perge[1,2]†

arXiv:2205.05087



Superconductivity in trilayer graphene without a moiré superlattice: theory

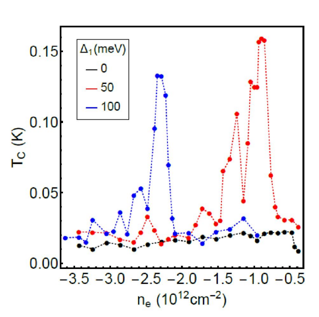

PHYSICAL REVIEW B **105**, 075432 (2022)

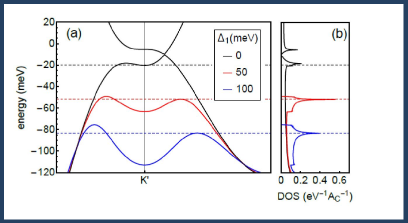
Superconductivity from repulsive interactions in rhombohedral trilayer graphene: A Kohn-Luttinger-like mechanism

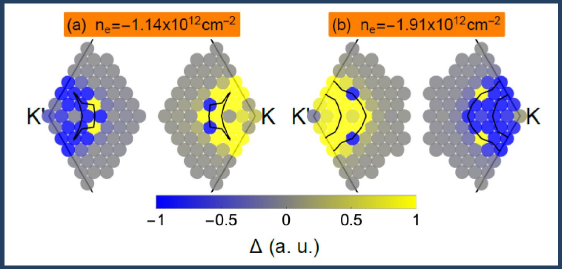
Tommaso Cea and Pierre A. Pantaleón
Imdea Nanoscience, Faraday 9, 28015 Madrid, Spain

Võ Tiến Phong
Department of Physics and Astronomy, University of Pennsylvania, Philadelphia PA 19104, USA

Francisco Guinea
Imdea Nanoscience, Faraday 9, 28015 Madrid, Spain;
Donostia International Physics Center, Paseo Manuel de Lardizábal 4, 20018 San Sebastián, Spain;
and Ikerbasque, Basque Foundation for Science, 48009 Bilbao, Spain





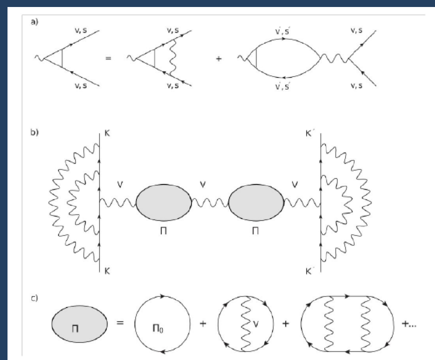


Superconductivity in bilayer graphene without a moiré superlattice: theory

Spin-triplet superconductivity at the onset of isospin order in biased bilayer graphene

Zhiyu Dong[1], Andrey V. Chubukov[2], Leonid Levitov[1]

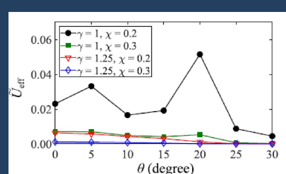
arXiv:2205.13353



Enhanced superconductivity through virtual tunneling in Bernal bilayer graphene coupled to WSe_2

Yang-Zhi Chou,[1,*] Fengcheng Wu,[2,3] and Sankar Das Sarma[1]

arXiv:2206.09922

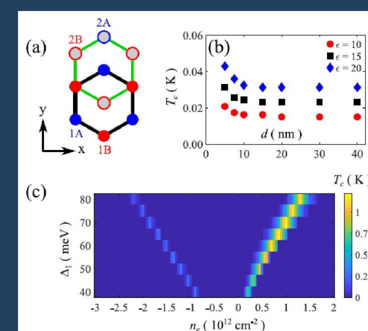


PHYSICAL REVIEW B **105**, L100503 (2022)

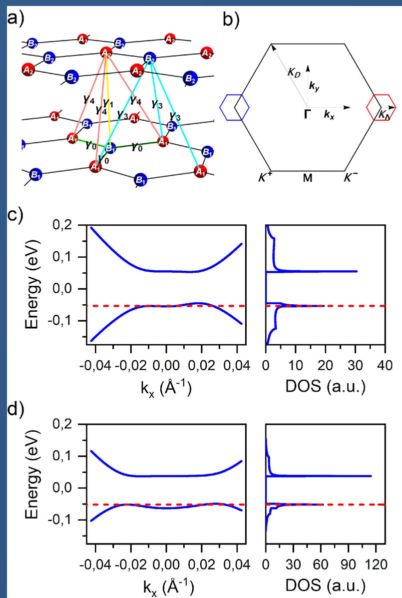
Letter

Acoustic-phonon-mediated superconductivity in Bernal bilayer graphene

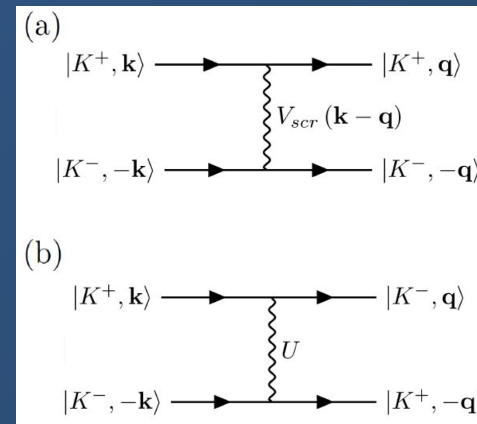
Yang-Zhi Chou,[1,*] Fengcheng Wu,[2] Jay D. Sau,[1] and Sankar Das Sarma[1]



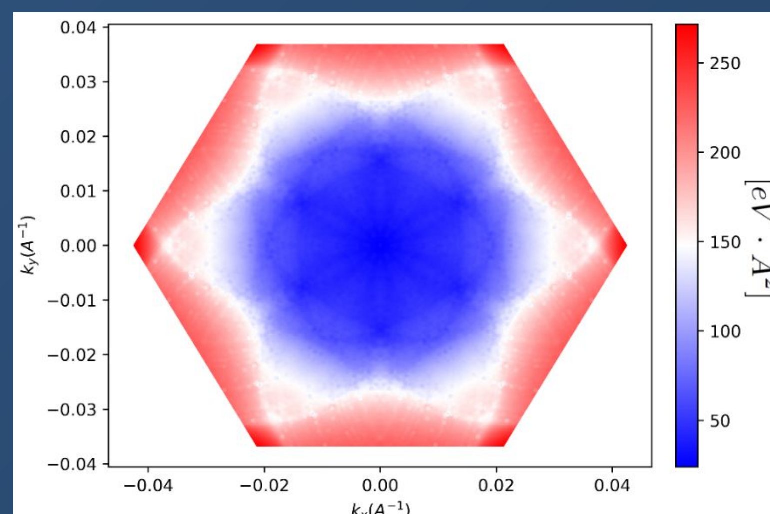
Superconductivity in bilayer graphene without a moiré superlattice: model



Continuum model

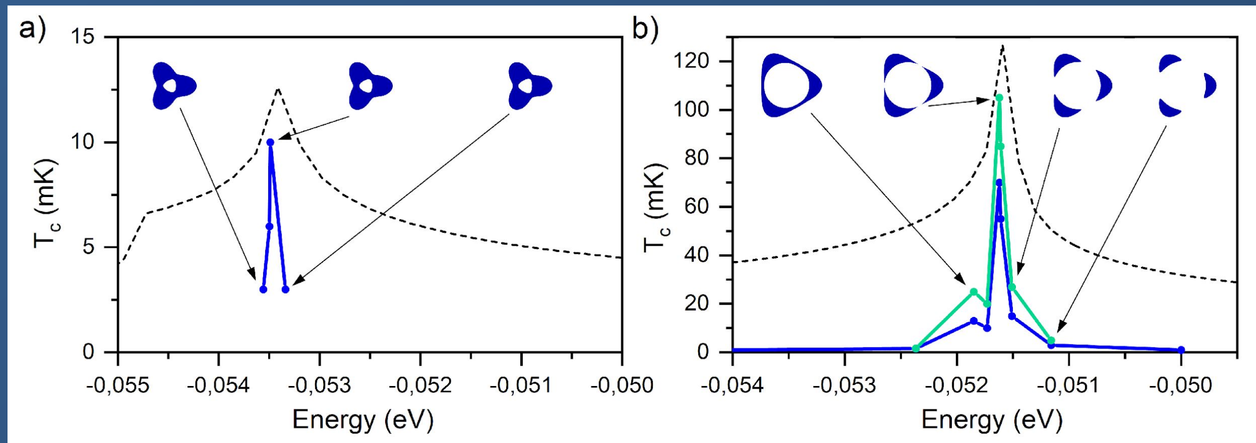


Pairing interaction

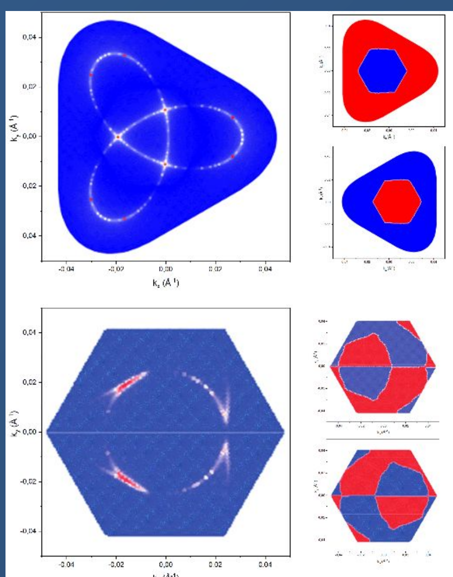


Screened interaction

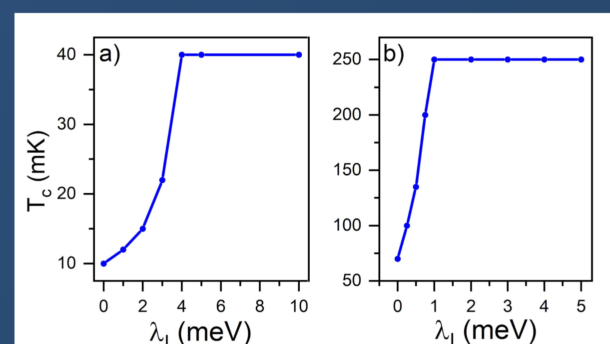
Superconductivity in bilayer graphene without a moiré superlattice: results



Fermi surface and critical temperature



Order parameter



Effect of spin orbit coupling

STM spectroscopy of the superconducting phase

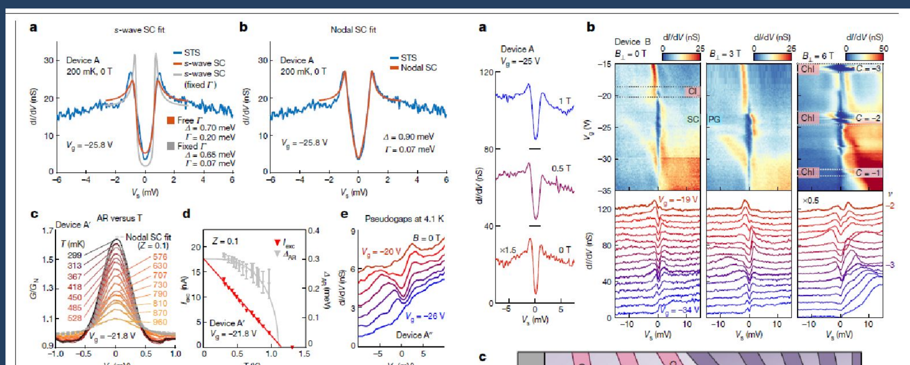
Article

Evidence for unconventional superconductivity in twisted bilayer graphene

https://doi.org/10.1038/s41586-021-04121-x

Received: 16 June 2021

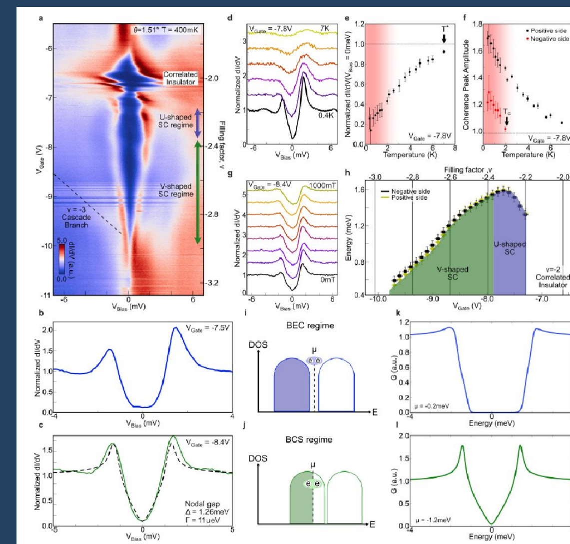
Myungchul Oh[1,4], Kevin P. Nuckolls[1,4], Dillon Wong[1,4], Ryan L. Lee[1], Xiaomeng Liu[1], Kenji Watanabe[2], Takashi Taniguchi[3] & Ali Yazdani[1✉]



Spectroscopic Signatures of Strong Correlations and Unconventional Superconductivity in Twisted Trilayer Graphene

Hyunjin Kim,[1,2,3,*] Youngjoon Choi,[1,2,3,*] Cyprian Lewandowski,[2,3,4] Alex Thomson,[2,3,4,5] Yiran Zhang,[1,2,3] Robert Polski,[1,2] Kenji Watanabe,[6] Takashi Taniguchi,[6] Jason Alicea,[2,3,4] and Stevan Nadj-Perge[1,2,†]

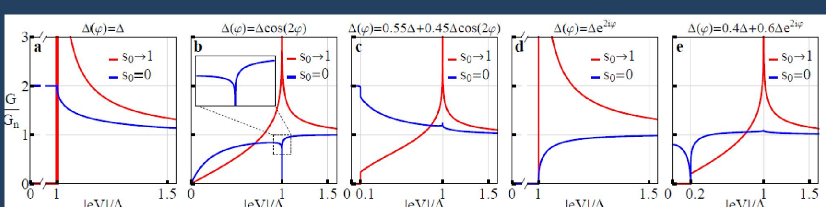
arXiv:2109.12127



Andreev reflection in scanning tunneling spectroscopy of unconventional superconductors

P. O. Sukhachov,[1,*] Felix von Oppen,[2] and L. I. Glazman[1]

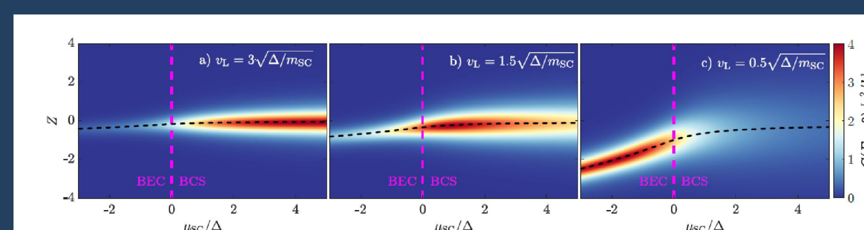
arXiv:2208.05979



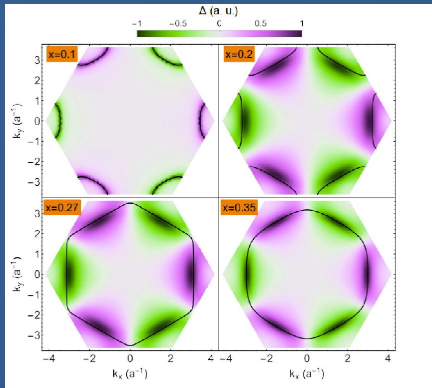
Andreev reflection spectroscopy in strongly paired superconductors

Cyprian Lewandowski,[1,2] Étienne Lantagne-Hurtubise,[1,2] Alex Thomson,[1,2,3,4] Stevan Nadj-Perge,[5,2] and Jason Alicea[1,2]

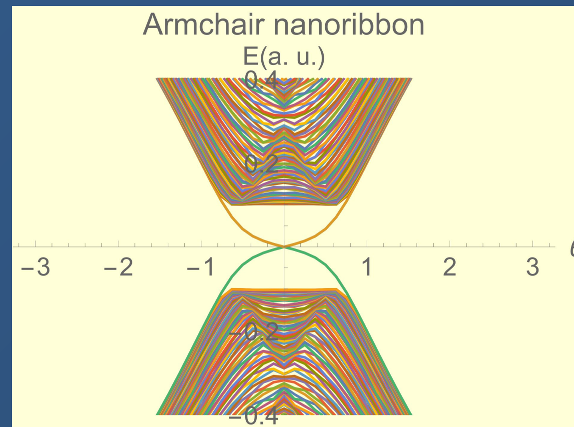
arXiv:2207.09494

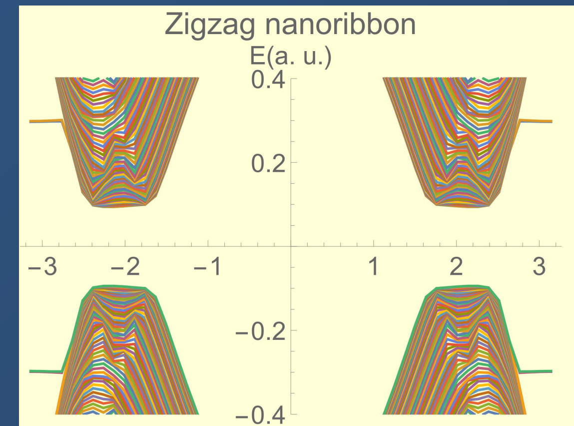


Edge modes in f-wave superconducting graphene



f-wave superconductivity. Constant gaps of opposite signs in the two valleys





Toy model: graphene + “Haldane” superconducting gap

PRL 111, 056402 (2013) PHYSICAL REVIEW LETTERS week ending 2 AUGUST 2013

Time-Reversal-Invariant Topological Superconductivity and Majorana Kramers Pairs

Fan Zhang,* C. L. Kane, and E. J. Mele

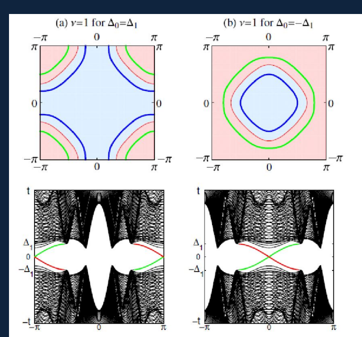
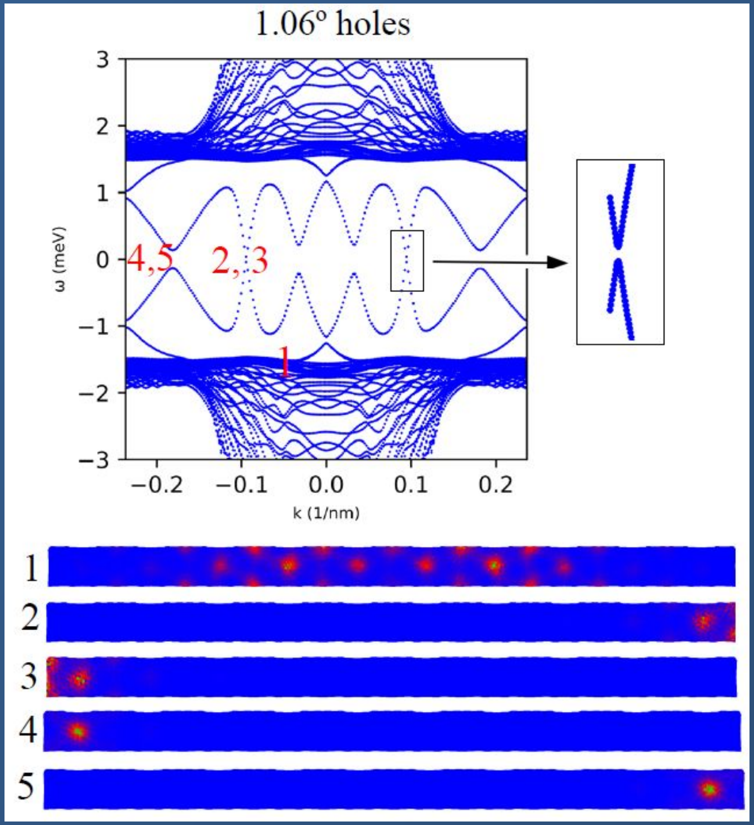
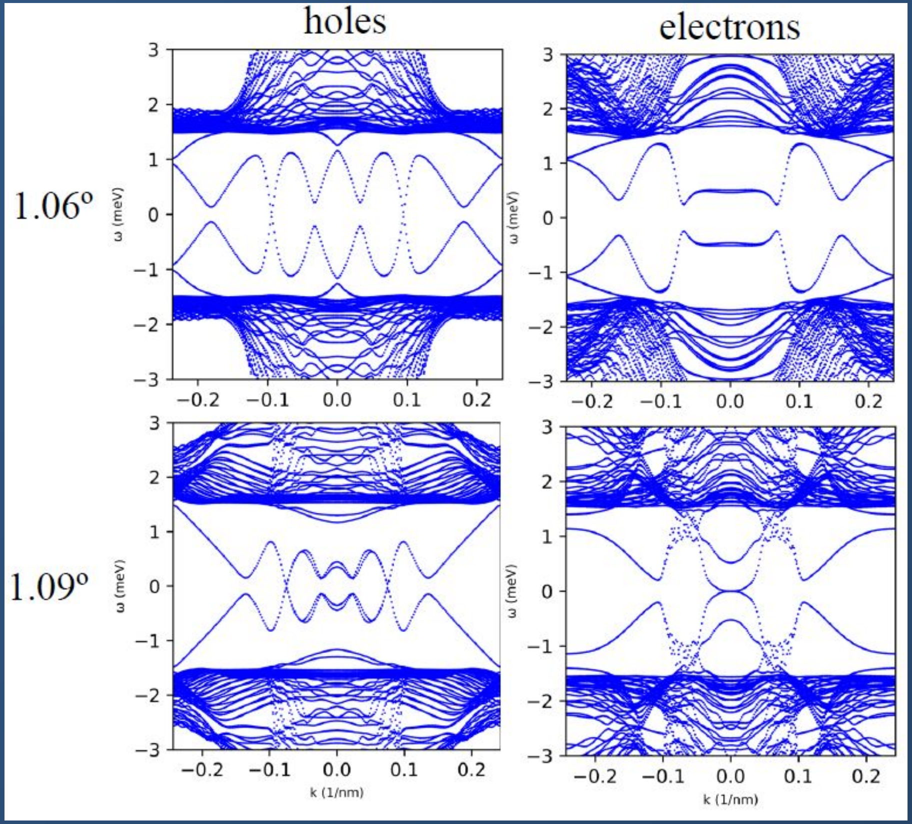


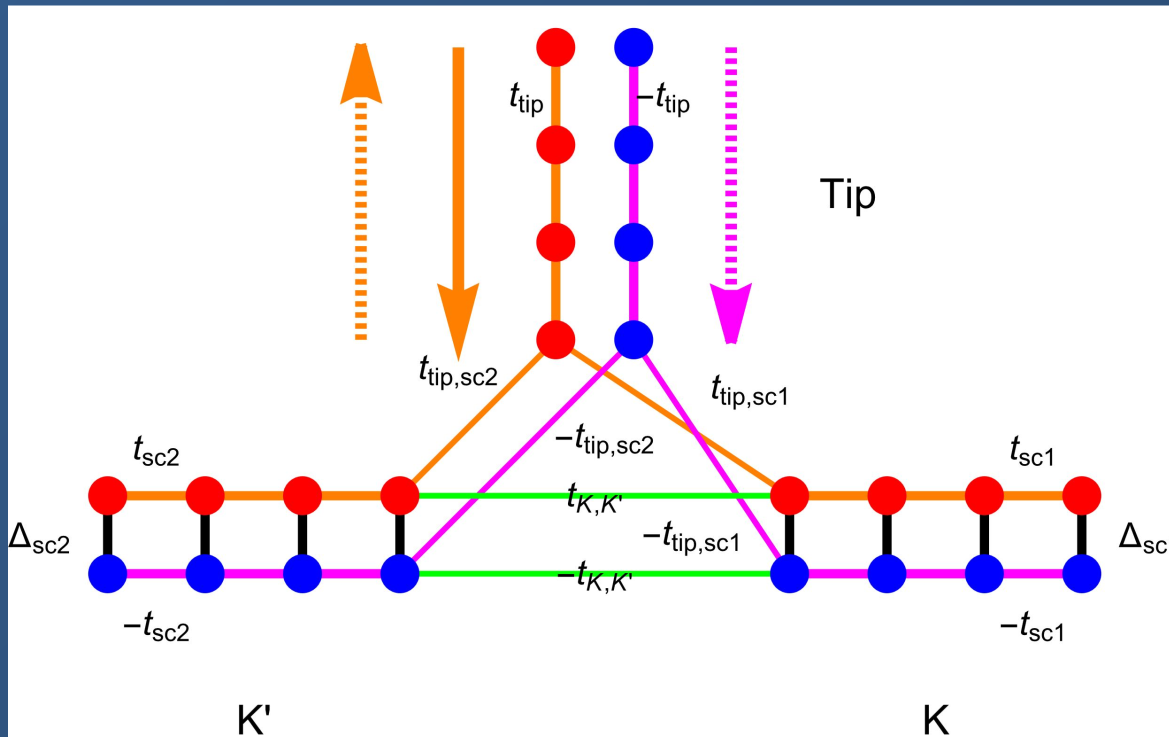
FIG. 2 (color online). Upper panels: the two Fermi surfaces (blue and green) of the single-particle bands for the $\nu = 1$ state; the closed nodal line (red) of Δ_k, separating two regions in which Δ_k has opposite signs. Lower panel: BdG spectrum of a 2D ribbon as a function of k for the $\nu = 1$ state with $\mu = \epsilon_0$. The red and green lines indicate the helical Majorana edge states. We choose parameter values $t = 10$, $\lambda_R = 5$, and $|\Delta_0| = \Delta_1 = 2$. (a) $\Delta_0 > 0$ and (b) $\Delta_0 < 0$.

Edge modes in f-wave superconducting twisted bilayer graphene





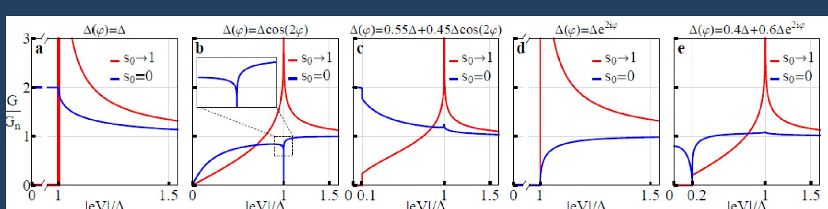
Tunneling and contact spectra in f-wave superconducting twisted bilayer graphene



Andreev reflection in scanning tunneling spectroscopy of unconventional superconductors

P. O. Sukhachov,[1,*] Felix von Oppen,[2] and L. I. Glazman[1]

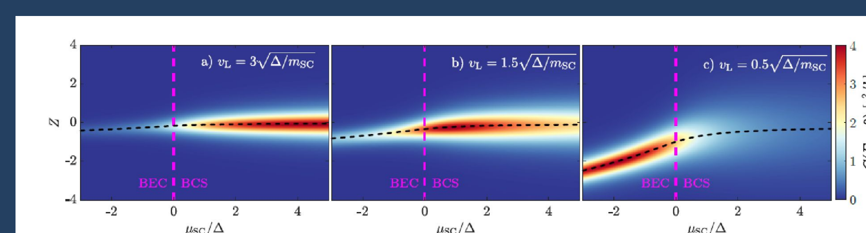
arXiv:2208.05979



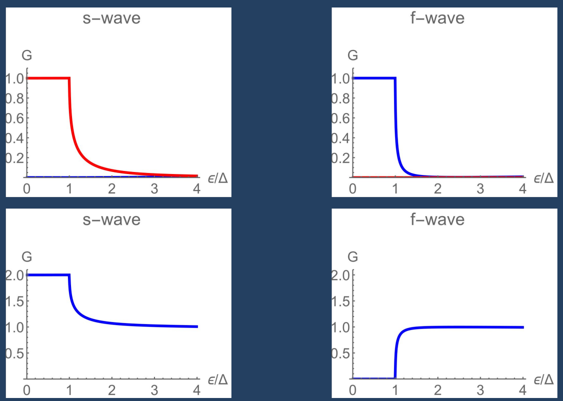
Andreev reflection spectroscopy in strongly paired superconductors

Cyprian Lewandowski,[1,2] Étienne Lantagne-Hurtubise,[1,2] Alex Thomson,[1,2,3,4] Stevan Nadj-Perge,[5,2] and Jason Alicea[1,2]

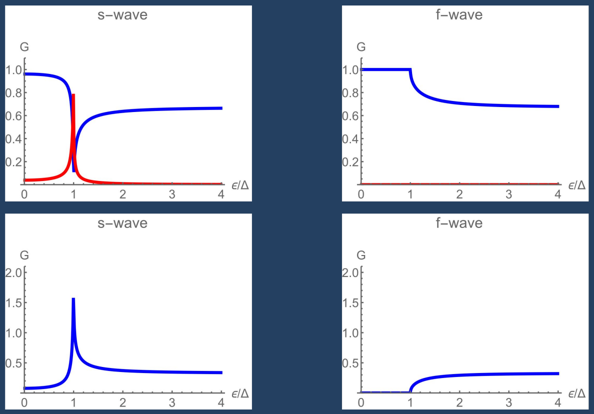
arXiv:2207.09494



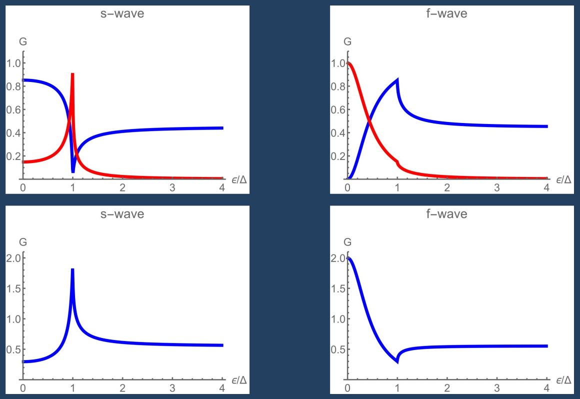
Tunneling and contact spectra in f-wave superconducting twisted bilayer graphene



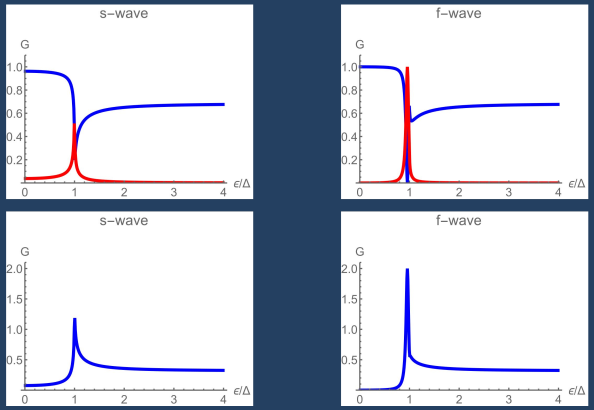
Contact regime: BTK limit



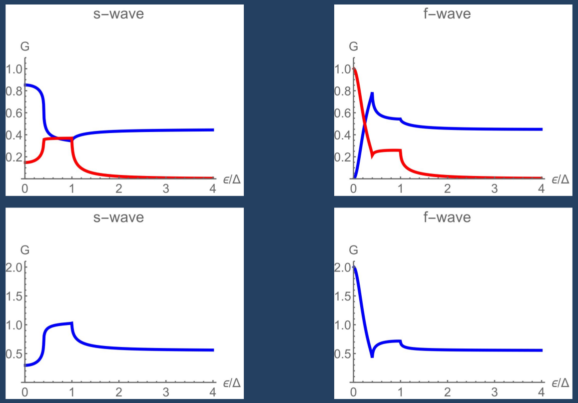
Contact regime: Fermi velocity mismatch



Contact regime: Intervalley scattering



Tunneling regime.



Contact regime: spin-orbit coupling

Electrons and phonons in twisted bilayer graphene

Niels R. Walet

Tommaso Cea
Pierre Pantaleón
José Silva-Guillén
Yago Ferreiros
Andreas Sinner
Héctor Sainz-Cruz
Alejandro Jimeno Pozo
Xueheng Kuang

Also:
V. T. Phong (U. Penn)
L. Levitov (MIT)
L. Fu (MIT)
V. Crépel (MIT)
S. Yuan (Wuhan U.)
J. Lischner, (Imperial C.)
Z. Goodwin, (Imperial C.)
H. Rostami (Nordita)

de Madrid

Twisted bilayer graphene almost aligned with hBN

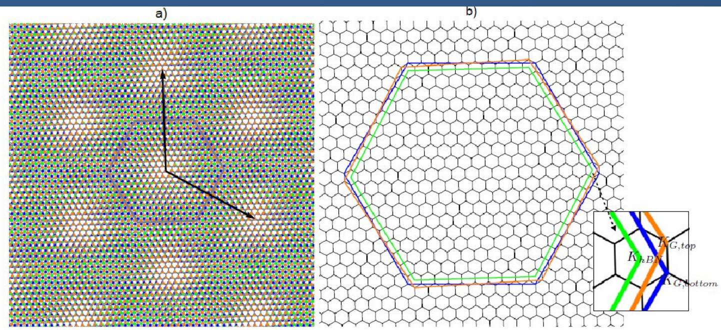


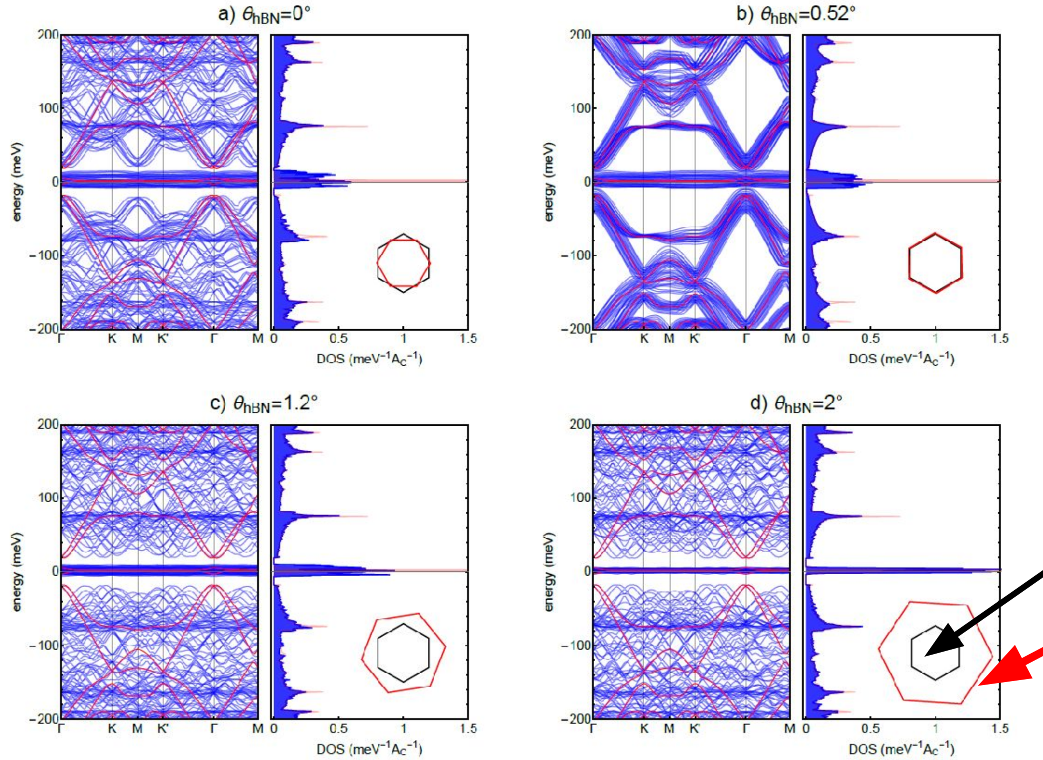
FIG. 1. a) Sketch of the moiré superlattice. The blue and orange points represent the carbon atoms, while the green points refer to the substrate. b) The large hexagons represent the BZs of the constituting layers. Their folding gives rise to the mini-BZs represented by the small black hexagons. In the inset: one side of the mini-BZ connects the corners of the BZs of each pair of layers.

PHYSICAL REVIEW B **102**, 155136 (2020)

Band structure of twisted bilayer graphene on hexagonal boron nitride

Tommaso Cea[1,2,*] Pierre A. Pantaleón[2,*] and Francisco Guinea[2,3]

- Details of the potential induced by the hBN substrate, beyond spatial averages, matter.
- Near the magic angles, the flat bands are significantly distorted.



Black hexagon:
TBG Brillouin Zone.
Red hexagon:
hBN-TBG Brillouin Zone

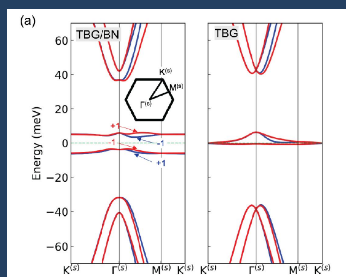
Related works

PHYSICAL REVIEW B **102**, 035441 (2020)

Symmetry breaking in the double moiré superlattices of relaxed twisted bilayer graphene on hexagonal boron nitride

Xianqing Lin[1,*] and Jun Ni[2]
[1]College of Science, Zhejiang University of Technology, Hangzhou 310023, People's Republic of China
[2]State Key Laboratory of Low-Dimensional Quantum Physics and Frontier Science Center for Quantum Information, Department of Physics, Tsinghua University, Beijing 100084, People's Republic of China

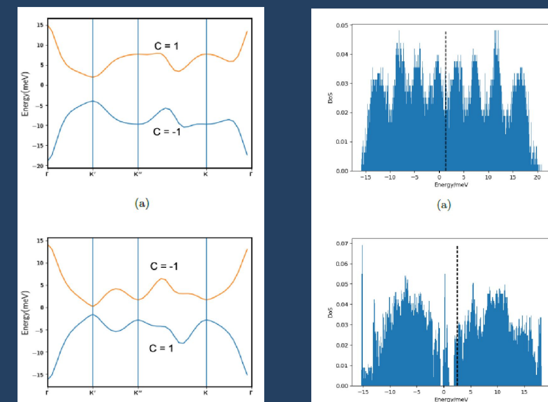
(Received 1 June 2020; revised 15 July 2020; accepted 16 July 2020; published 29 July 2020)



Quasiperiodicity, band topology, and moiré graphene

Dan Mao and T. Senthil

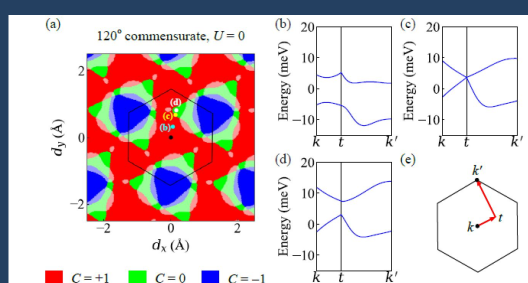
Phys. Rev. B **103**, 115110 (2021)



Moiré Commensurability and the Quantum Anomalous Hall Effect in Twisted Bilayer Graphene on Hexagonal Boron Nitride

Jingtian Shi, Jihang Zhu, and A.H. MacDonald

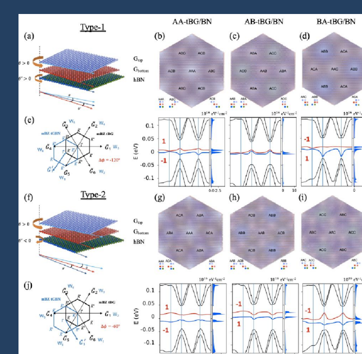
Phys. Rev. B **103**, 075122 (2021)



Electron-hole asymmetry and band gaps of commensurate double moire patterns in twisted bilayer graphene on hexagonal boron nitride

Jiseon Shin, Youngju Park, and Bheema Lingam Chittari
Department of Physics, University of Seoul, Seoul 02504, Korea
Jeil Jung*

Phys. Rev. B **103**, 075423 (2021)

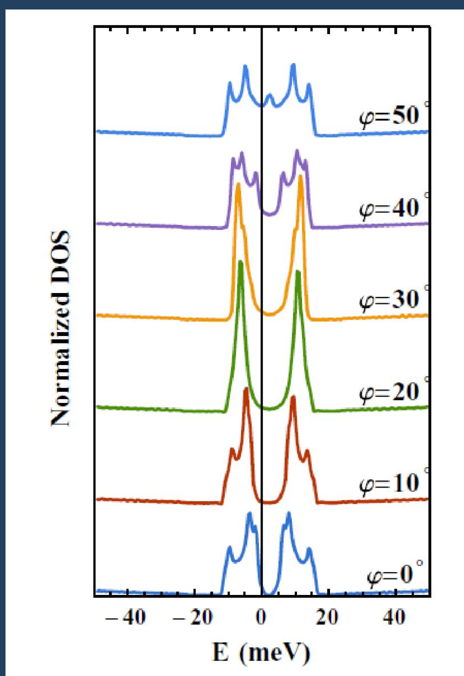


Narrow bands and strains

PHYSICAL REVIEW B **100**, 035448 (2019)

Designing flat bands by strain

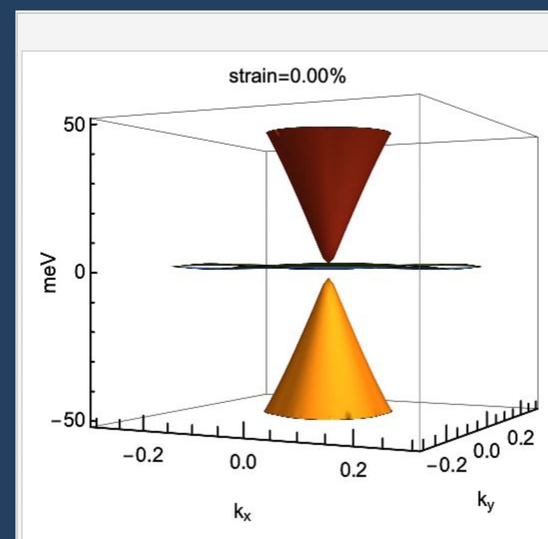
Zhen Bi,* Noah F. Q. Yuan, and Liang Fu



PHYSICAL REVIEW LETTERS **127,** 126405 (2021)

Heterostrain Determines Flat Bands in Magic-Angle Twisted Graphene Layers

Florie Mesple,[1,*] Ahmed Missaoui,[2] Tommaso Cea,[3,4] Loic Huder,[5] Francisco Guinea,[3,6] Guy Trambly de Laissardière,[2] Claude Chapelier,[1] and Vincent T. Renard[1,†]



Non linear Hall effect is metals with non trivial bands

PHYSICAL REVIEW B **92**, 235447 (2015)

Topological currents in black phosphorus with broken inversion symmetry

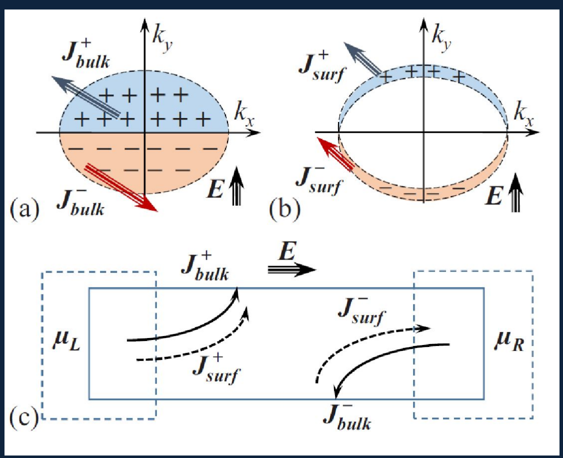
Tony Low,[1] Yongjin Jiang,[1,2] and Francisco Guinea[3,4]

PRL **115**, 216806 (2015) PHYSICAL REVIEW LETTERS 20 N

Quantum Nonlinear Hall Effect Induced by Berry Curvature Dipole in Time-Reversal Invariant Materials

Inti Sodemann and Liang Fu

- The Hall effect requires the breaking of time reversal symmetry and bands with non trivial topology.
- A stationary current breaks time reversal.
- Materials with non trivial bands show Hall currents at high electric fields.



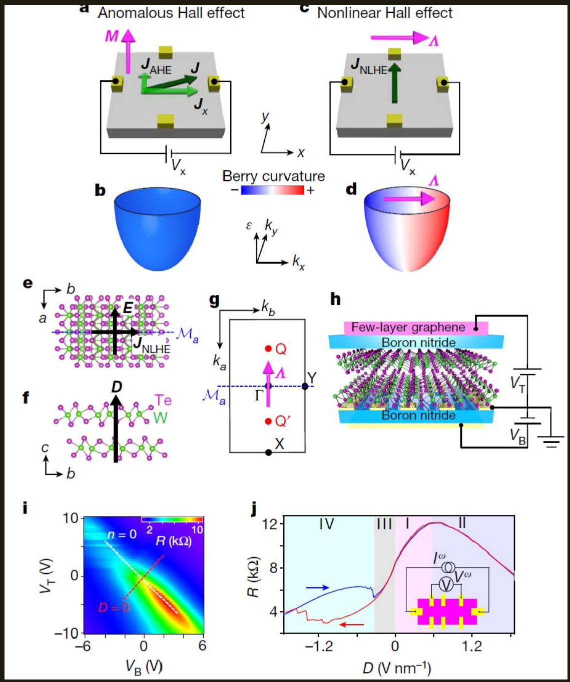
LETTER

https://doi.org/10.1038/s41586-018-0807-6

Observation of the nonlinear Hall effect under time-reversal-symmetric conditions

Qiong Ma[1,13], Su-Yang Xu[1,13], Huitao Shen[1,13], David MacNeill[1], Valla Fatemi[1], Tay-Rong Chang[2], Andrés M. Mier Valdivia[1], Sanfeng Wu[1], Zongzheng Du[3,4,5], Chuang-Han Hsu[6,7], Shiang Fang[8], Quinn D. Gibson[9], Kenji Watanabe[10], Takashi Taniguchi[10], Robert J. Cava[9], Efthimios Kaxiras[8,11], Hai-Zhou Lu[3,4], Hsin Lin[12], Liang Fu[1], Nuh Gedik[1]* & Pablo Jarillo-Herrero[1]*

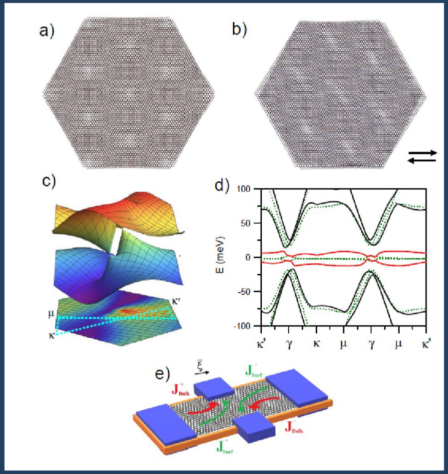
Nature **565**, 337 (2019)



Tunable large Berry dipole in strained twisted bilayer graphene

Pierre A. Pantaleón[1,*] Tony Low[2], and Francisco Guinea[1,3]

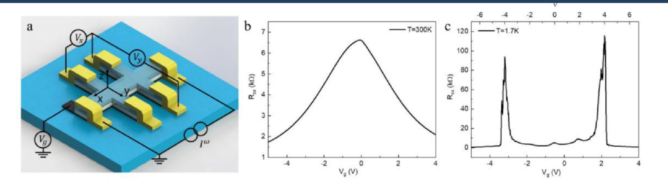
Phys. Rev. B **103**, 205403 (2021)

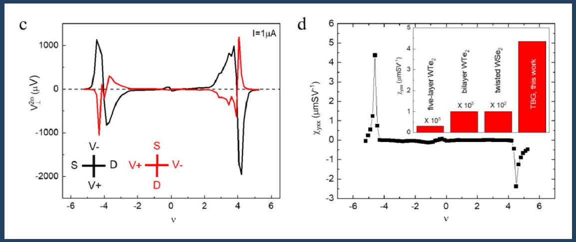


Giant second-order nonlinearity of chiral Bloch electrons in twisted bilayer graphene

Junxi Duan[1,2], Yu Jian[1,2], Yang Gao[3], Huimin Peng[1,2], Jinrui Zhong[1,2], Qi Feng[1,2], Yugui Yao[1,2]

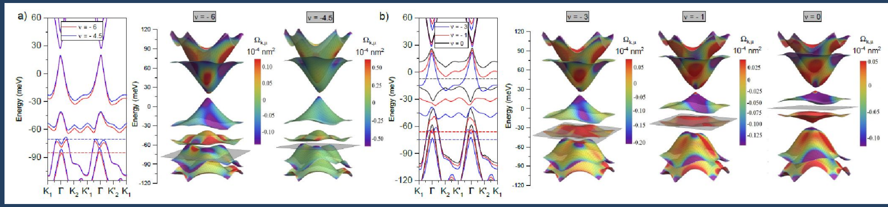
arXiv:2201.09274





Interaction-Enhanced Topological Hall Effects in Strained Twisted Bilayer Graphene

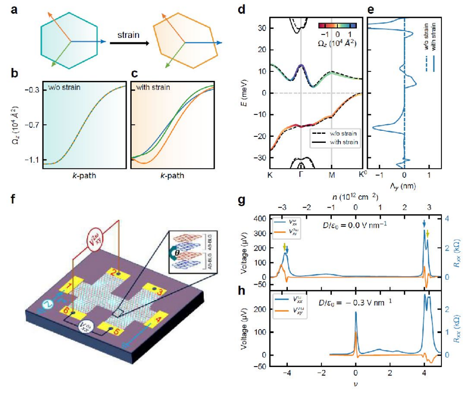
Pierre A. Pantaleón,[1,*] Võ Tiến Phong,[2] Gerardo G. Naumis,[3] and Francisco Guinea[1,4]



Berry curvature dipole senses topological transition in a moiré superlattice

Subhajit Sinha[1†*], Pratap Chandra Adak[1†], Atasi Chakraborty[2], Kamal Das[2], Koyendrila Debnath[3], L. D. Varma Sangani[1], Kenji Watanabe[4], Takashi Taniguchi[5], Umesh V. Waghmare[3], Amit Agarwal[2*], and Mandar M. Deshmukh[1*]

arXiv:2204.02848



Twisted (chiral) phonons

PHYSICAL REVIEW B **75**, 045404 (2007)

Electron-phonon coupling and Raman spectroscopy in graphene

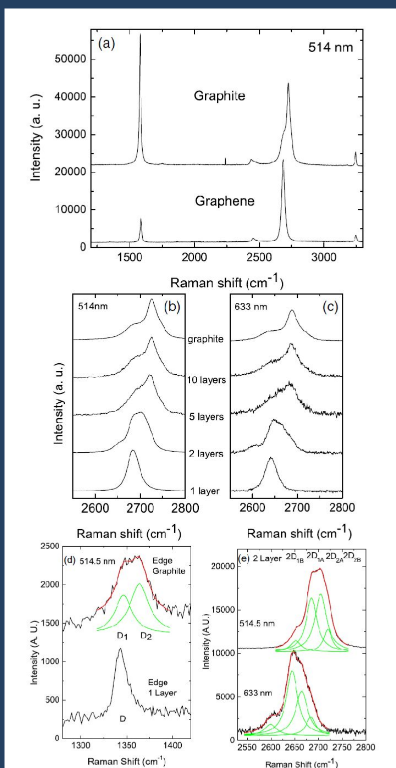
A. H. Castro Neto
nent of Physics, Boston University, 590 Commonwealth Avenue, Boston, Massachusetts 02215, USA

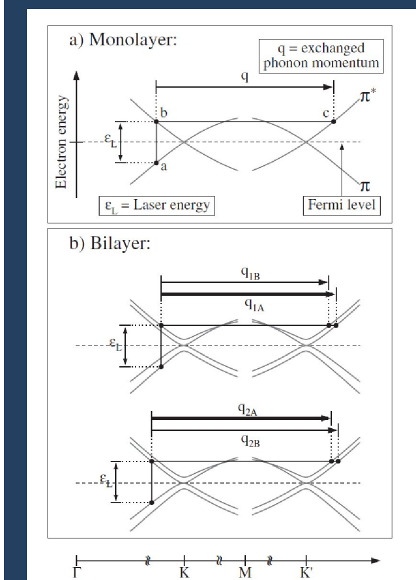
Francisco Guinea

PRL **97,** 187401 (2006) PHYSICAL REVIEW LETTERS week ending 3 NOVEMBER 2006

Raman Spectrum of Graphene and Graphene Layers

A. C. Ferrari,[1,*] J. C. Meyer,[2] V. Scardaci,[1] C. Casiraghi,[1] M. Lazzeri,[3] F. Mauri,[3] S. Piscanec,[1] D. Jiang,[4] K. S. Novoselov,[4] S. Roth,[2] and A. K. Geim[4]



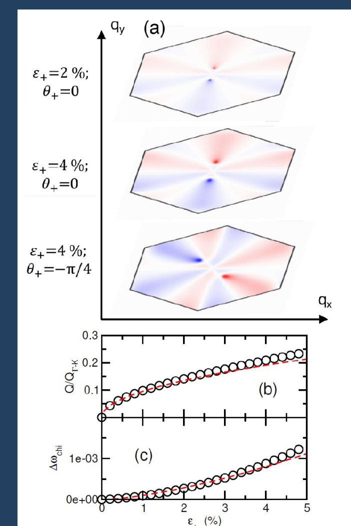


K phonons in graphene stacks

Strain-driven chiral phonons in two-dimensional hexagonal materials

Habib Rostami,[1] Francisco Guinea,[2,3,4] and Emmanuele Cappelluti[5]

arXiv:2022.04909



Twisted (chiral) phonons, theory

PHYSICAL REVIEW X **9**, 041010 (2019)

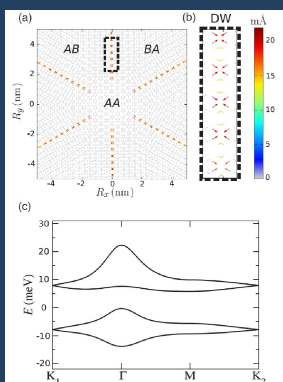
Valley Jahn-Teller Effect in Twisted Bilayer Graphene

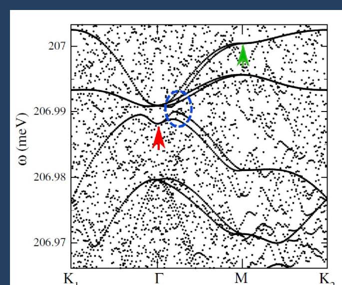
M. Angeli,[1] E. Tosatti,[1,2,3] and M. Fabrizio[1]

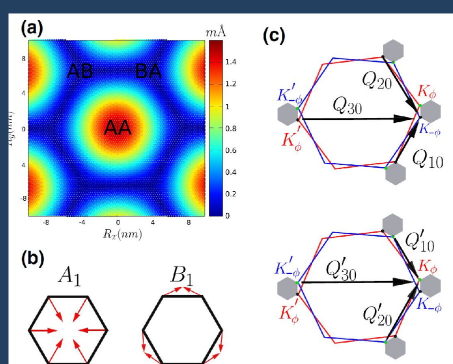
Eur. Phys. J. Plus (2020) 135:630
https://doi.org/10.1140/epjp/s13360-020-00647-7
THE EUROPEAN PHYSICAL JOURNAL
Regular Article

Jahn–Teller coupling to moiré phonons in the continuum model formalism for small-angle twisted bilayer graphene

Mattia Angeli[a], Michele Fabrizio

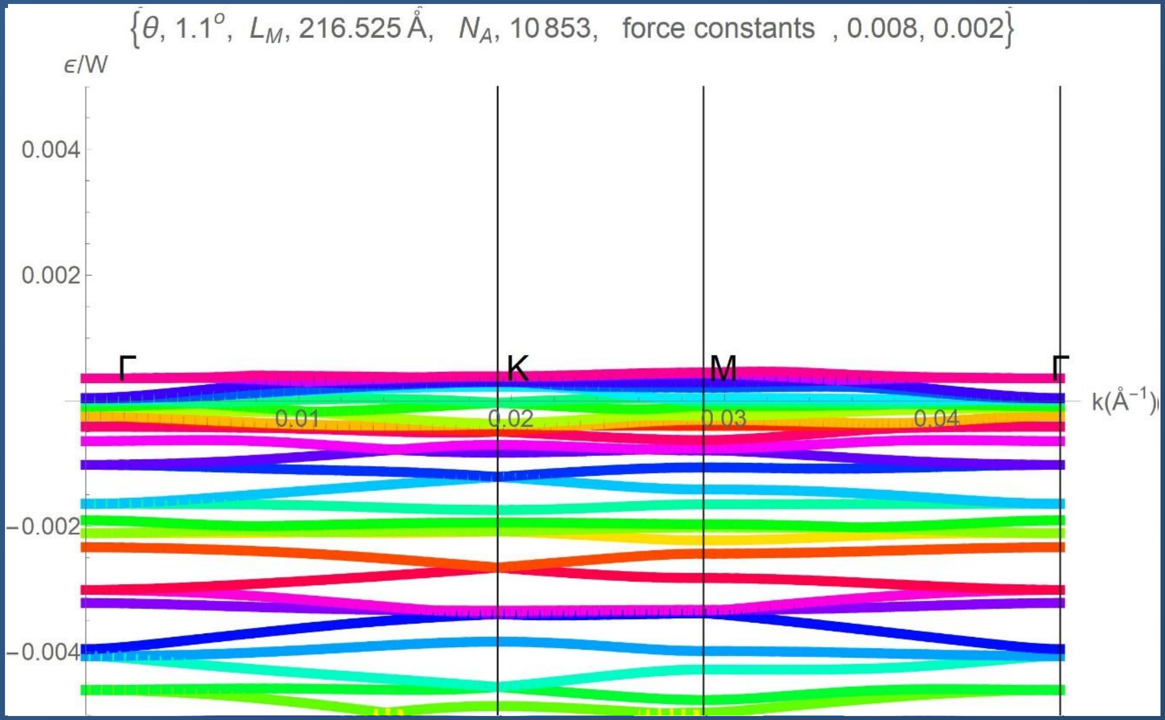






- Strong coupling to phonons
- Coupling to valley degree of freedom
- Flat folded bands

Twisted phonons, magic angles

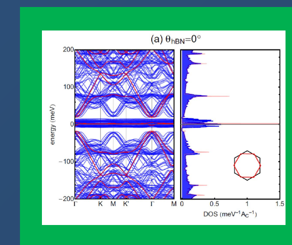


- Coupled Dirac phonons in a twisted background
- Hamiltonian similar to the electronic Hamiltonian
- Multiple interference processes
- Similar phonons in twisted hBN and twisted TMD's

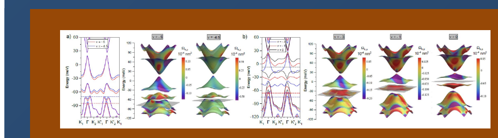
Electrons and phonons in twisted bilayer graphene

- **The largest interaction in twisted graphene bilayers near magical angles is the long range Coulomb interaction. Away from the neutrality point, the inhomogeneous distribution of carriers leads to an electrostatic potential.**

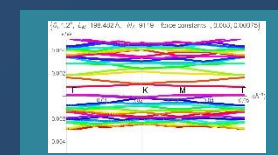
$$\frac{e^2}{\epsilon L_M} \approx 10 - 40 \text{ meV}$$



- **The narrow bands are very fragile, and they are substantially changed by interactions, the substrate, strains, …**

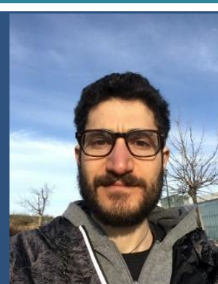


- **The non trivial topology of the bands enhance non linear effects, like the non linear Hall effect.**



- **There are also narrow phonon bands with non trivial topology.**

Niels R. Walet



Tomasso Cea

Pierre Pantaleon

Rory Brown

Also:
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L. Levitov (MIT)
L. Fu (MIT)
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- Twisted bilayer graphene is a unique material
- There is a large number of open problems

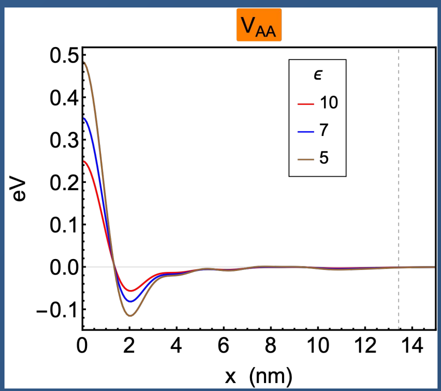
Long range interactions and superconductivity: the Kohn-Luttinger mechanism

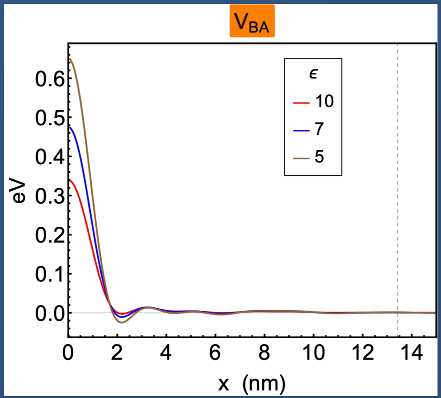
Some results which describe pairing channels using perturbative/diagrammatic analyses of the Coulomb interaction

- H. Isobe, N. F. Q. Yuan, and L. Fu, ***Unconventional superconductivity and density waves in twisted bilayer graphene***, Phys. Rev. X **8**, 041041 (2018).
- Y. Sherkunov and J. J. Betouras, ***Electronic phases in twisted bilayer graphene at magic angles as a result of van hove singularities and interactions***, Phys. Rev. B **98**, 205151 (2018).
- J. González and T. Stauber, ***Kohn-Luttinger superconductivity in twisted bilayer graphene***, Phys. Rev. Lett. **122**, 026801 (2019).
- B. Roy and V. Juricic, ***Unconventional superconductivity in nearly flat bands in twisted bilayer graphene***, Physical Review B **99**, 12 1407 (2019).
- D. V. Chichinadze, L. Classen, and A. V. Chubukov, ***Nematic superconductivity in twisted bilayer graphene***, Phys. Rev. B **101**, 224513 (2020).
- Y.-P. Lin and R. M. Nandkishore, ***Parquet renormalization group analysis of weak-coupling instabilities with multiple high-order van hove points inside the Brillouin zone***, Phys. Rev. B **102**, 245122 (2020).
- C. Lewandowski, D. Chowdhury, and J. Ruhman, ***Pairing in magic-angle twisted bilayer graphene: role of phonon and plasmon umklapp***, (2020), arXiv:2007.15002.
- W. Qin, B. Zou, and A. H. MacDonald, ***Critical magnetic fields and electron-pairing in magic-angle twisted bilayer graphene***, (2021), arXiv:2102.10504.
- C. Lewandowski, S. Nadj-Perge, and D. Chowdhury, ***Does filling-dependent band renormalization aid pairing in twisted bilayer graphene?***, (2021), arXiv:2102.05661.

to long-range effects.

Static screened potential

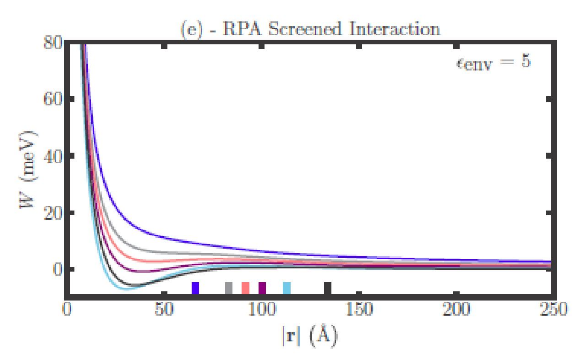




PHYSICAL REVIEW B **100**, 235424 (2019)

Attractive electron-electron interactions from internal screening in magic-angle twisted bilayer graphene

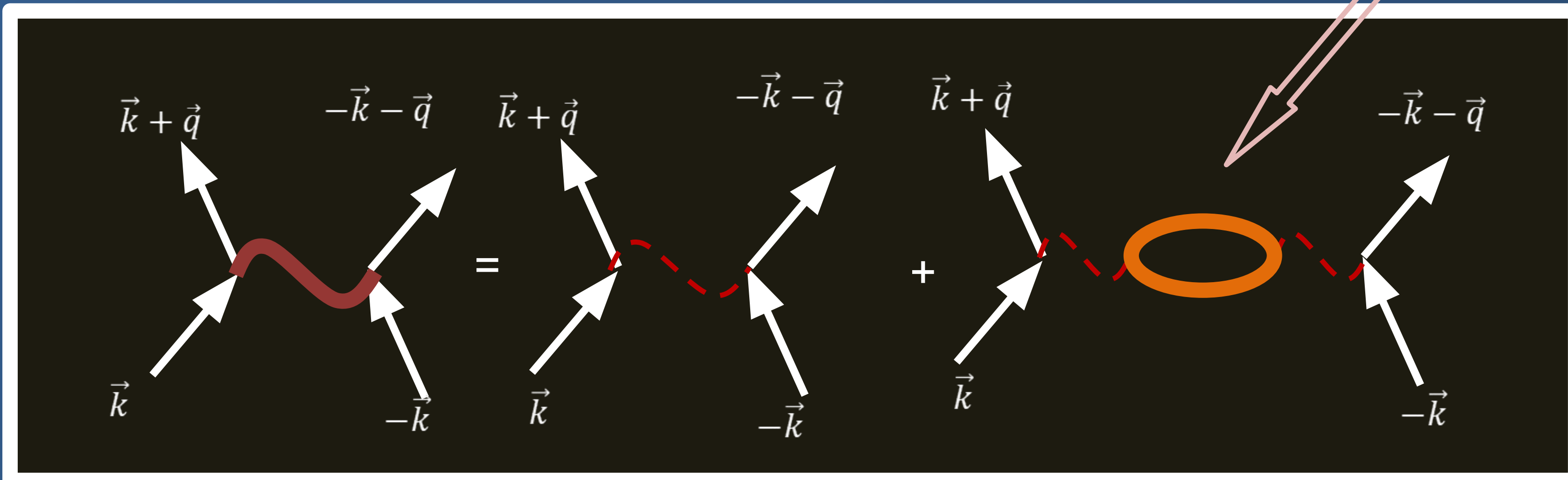
Zachary A. H. Goodwin, Fabiano Corsetti, Arash A. Mostofi, and Johannes Lischner



- Effective attraction at distances smaller than the size of the unit cell

Pairing interaction

RPA resummation



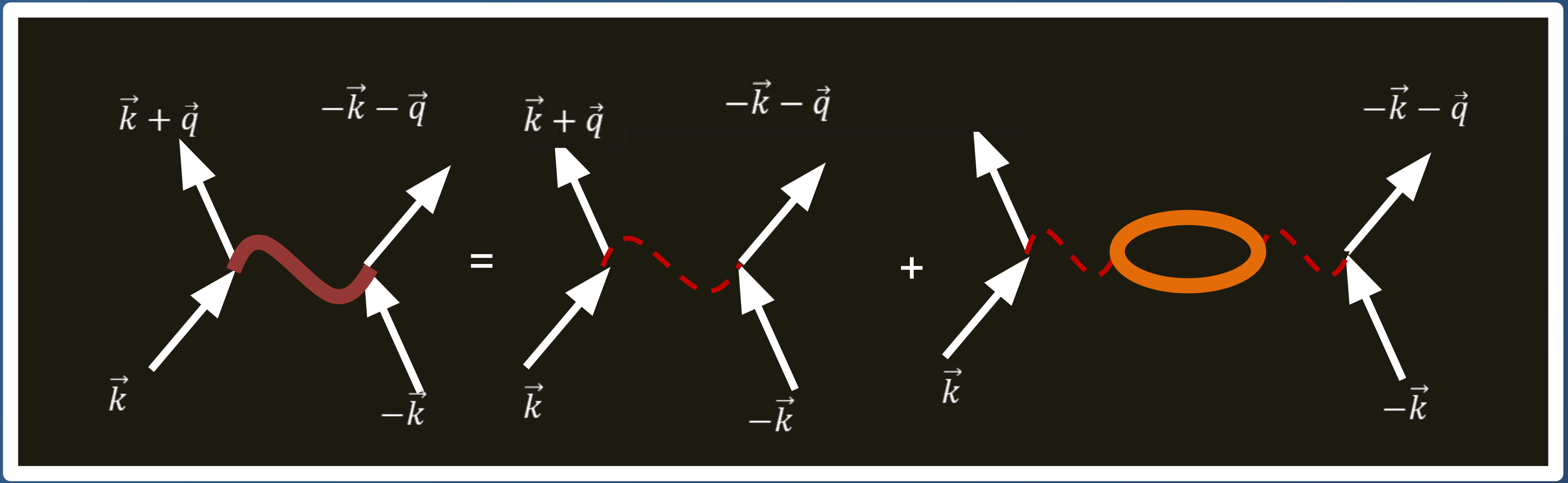
$$\widetilde{\Delta}^{m_1,m_2}_{\alpha,\beta}\left(\vec{k}\right) = \sum_{n_1,n_2,\vec{q}} \Gamma^{m_1,m_2}_{n_1,n_2,\alpha,\beta}\left(\vec{k},\vec{k}+\vec{q}\right)\widetilde{\Delta}^{n_1,n_2}_{\alpha,\beta}\left(\vec{k}+\vec{q}\right)$$

$$\mathcal{M}_{\vec{G}}\left(\vec{k},\vec{k}+\vec{q}\right) = \int d^2\vec{r}\, u^*_{\vec{k}}(\vec{r})e^{i\vec{G}\vec{r}}u_{\vec{k}+\vec{q}}(\vec{r})$$

$$\Gamma^{m_1,m_2}_{n_1,n_2,\alpha,\beta}\left(\vec{k},\vec{k}+\vec{q}\right) = -\frac{1}{\Omega}\sum_{\vec{G}_1,\vec{G}'_1}\sum_{\vec{G}_2,\vec{G}'_2}\sum_{i_1,i_2} \mathcal{V}^{scr}_{\vec{G}_1-\vec{G}'_1,\vec{G}_2-\vec{G}'_2}(\vec{q})\,\mathcal{M}^*_{\vec{G}_1-\vec{G}'_1}\left(\vec{k},\vec{k}+\vec{q}\right)\mathcal{M}_{\vec{G}_2-\vec{G}'_2}\left(\vec{k},\vec{k}+\vec{q}\right)$$

$$\times\sqrt{\frac{f\left(-E_{m_2,-\vec{k},\beta}+\mu\right)-f\left(E_{m_1,\vec{k},\alpha}-\mu\right)}{E_{m_2,-\vec{k},\beta}+E_{m_1,\vec{k},\alpha}-2\mu}}\sqrt{\frac{f\left(-E_{n_2,-\vec{k}-\vec{q},\beta}+\mu\right)-f\left(E_{n_1,\vec{k}+\vec{q},\alpha}-\mu\right)}{E_{m_2,-\vec{k}-\vec{q},\beta}+E_{m_1,\vec{k}+\vec{q},\alpha}-2\mu}}$$

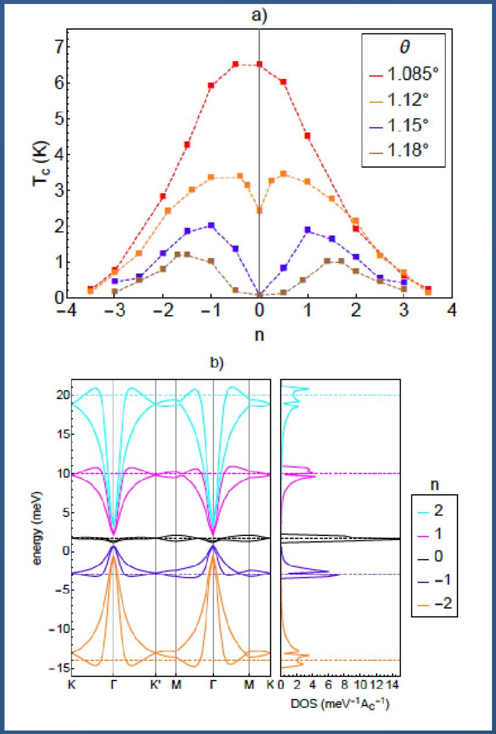
The calculations



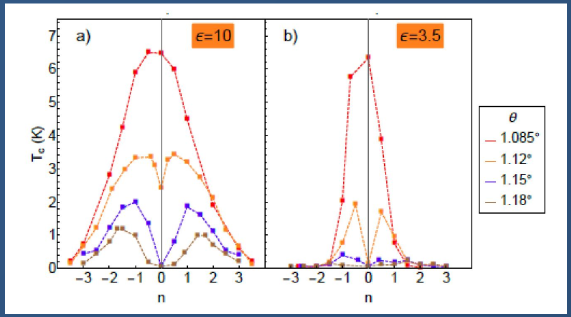
- Pairing between electrons in different valleys.
- Consistent with either spin singlet/valley triplet or spin triplet/valley singlet superconductivity.
- Bands calculated using the Hartree approximation.
- Instantaneous interactions.
- Convergent results as function of number of bands, and of the number of Umklapp processes.

Results: critical temperature

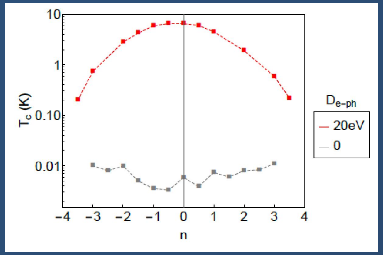
Critical temperatures



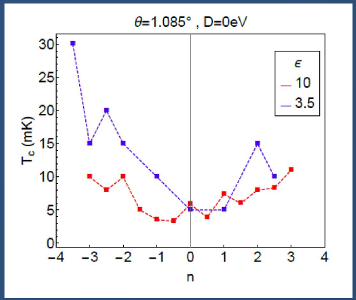
Bands, densities of states, magic angle



Dependence on dielectric constant



Critical temperature with and without e-ph interaction



Critical temperature for e-e interaction only

Superconductivity in a doped valley coherent insulator in magic angle graphene: Goldstone-mediated pairing and Kohn-Luttinger mechanism

Vladyslav Kozii,[1,2] Michael P. Zaletel,[1,2] and Nick Bultinck[1,3]

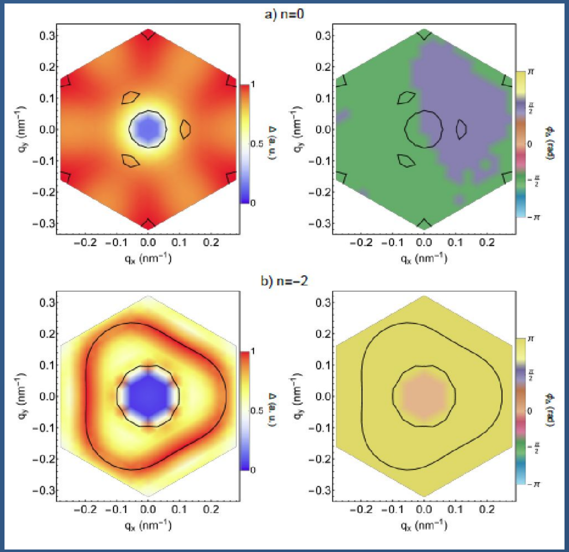
arXiv:2005.12961

Using $\varepsilon_F = 3.2$ meV and $\lambda \approx 0.08$, one finds $T_c \approx 1.3 \times 10^{-4}$ K, which is too low compared to the experimental values of $T_c \approx 0.3$ K. However, because of the

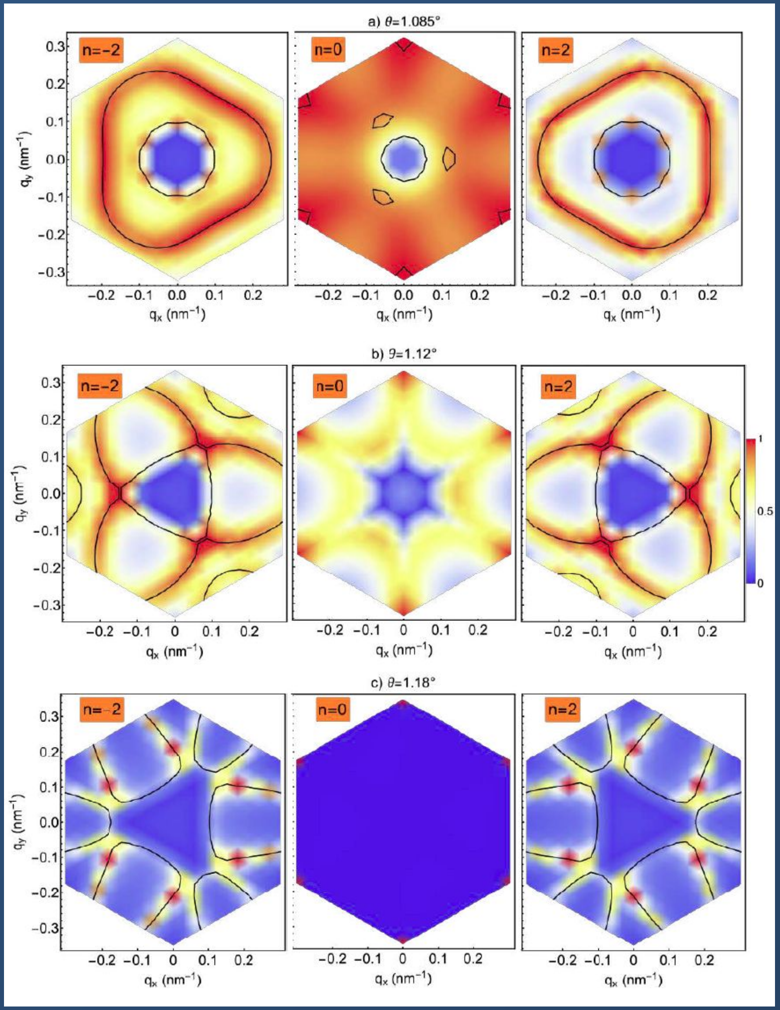
- The critical temperature is significantly enhanced by the electron-phonon interaction.
- Superconductivity correlates with the density of states at the Fermi level.
- The effect of external screening depends on the strength of the electron-phonon interaction.

Results: order parameter

Absolute magnitude Phase



Magic angle



Angle dependence

- At the magic angle, all states contribute to the order parameter
- For other angles, the order parameter is localized near the Fermi surface.
- The order parameter takes different values in different pockets.

Nature of the pairing interaction

Electronic susceptibility:

$$\chi_{\vec{G},\vec{G}'}(\vec{q},\omega) = \sum_{\alpha,\beta,\vec{k}} \left[\mathcal{M}_{\vec{G}}^{\alpha,\beta}\left(\vec{k},\vec{k}+\vec{q}\right)\right]^* \mathcal{M}_{\vec{G}'}^{\alpha,\beta}\left(\vec{k},\vec{k}+\vec{q}\right) \frac{n_{\vec{k}+\vec{q}}^{\beta} - n_{\vec{k}}^{\alpha}}{\omega - \epsilon_{\vec{k}+\vec{q}}^{\beta} + \epsilon_{\vec{k}}^{\alpha}}$$

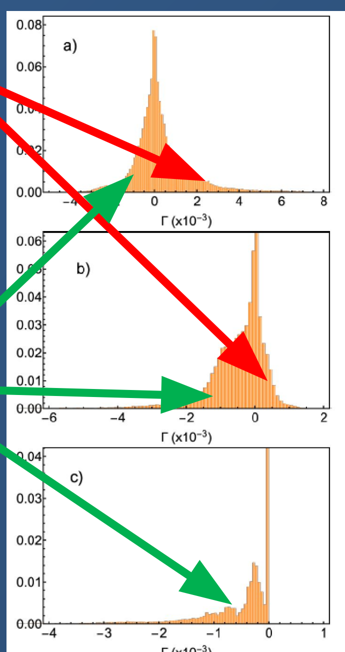
Form factor: $\mathcal{M}_{\vec{G}}^{\alpha,\beta}\left(\vec{k},\vec{k}+\vec{q}\right) = \left\langle \vec{k},\alpha \middle| e^{i\vec{G}\vec{r}} \middle| \vec{k}+\vec{q},\beta \right\rangle$

Superconducting kernel: $\Gamma_{\alpha,\beta}\left(\vec{k},\vec{k}+\vec{q}\right)$

- Umklapp processes are crucial.
- Form factors lead to attractive interactions.
- The order parameter does not change sign.
- Consistent with spin singlet/valley triplet or spin triplet/valley singlet superconductivity.

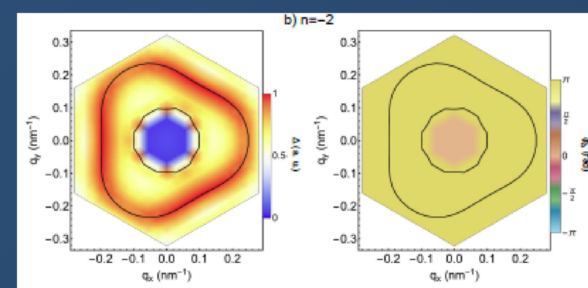
Attraction

Repulsion



Distribution of the matrix elements of the superconducting kernel

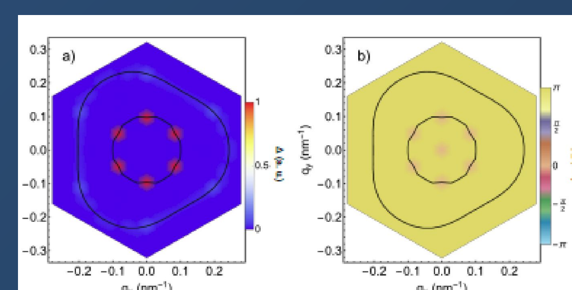
Including phonons



Excluding phonons

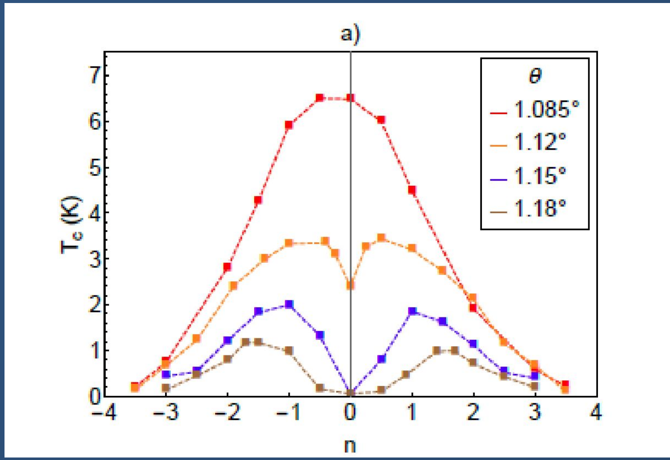
Excluding phonons and simplifying the form factors

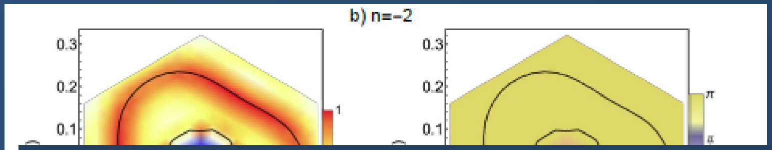
$$\mathcal{M}_{\vec{G}}^{\alpha,\beta}\left(\vec{k},\vec{k}+\vec{q}\right) \approx \delta_{\alpha,\beta}\, g\left(\vec{k},\vec{k}+\vec{q}\right) f_{\vec{G}}$$



Other effects

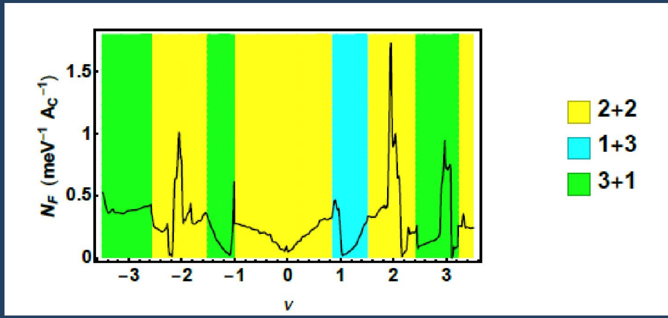
- Transverse acoustical and optical phonons are not included.
  Possible enhancement of T_c.
- No exchange effects. Spin and/or valley polarized phases not considered.
  Calculation approximately correct for spin polarized phases with equal occupancy of the K and K' valleys, such as the 2+2 phase near ν=2.
  No soft spin and/or valley modes.
- No retardation effects.
  Upper bound on the critical temperature,
  $k_B T_c \leq \hbar\omega_{ph}$.





Superconducting properties

Mechanism intrinsic to twisted bilayer graphene.
Multigap superconductor.
No sign changes in the order parameter within each valley.
Weak pair breaking due to elastic scattering.



Superconductivity in twisted trilayers

Article

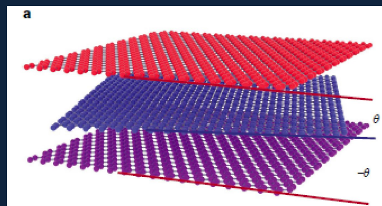
Tunable strongly coupled superconductivity in magic-angle twisted trilayer graphene

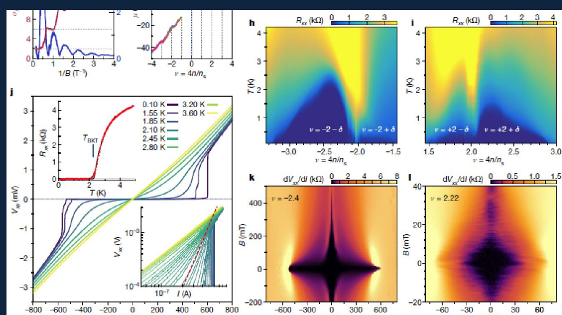
https://doi.org/10.1038/s41586-021-03192-0

Jeong Min Park[1,4], Yuan Cao[1,4], Kenji Watanabe[2], Takashi Taniguchi[3] & Pablo Jarillo-Herrero[1]

Received: 26 October 2020

Nature | Vol 590 | 11 February 2021 | 249



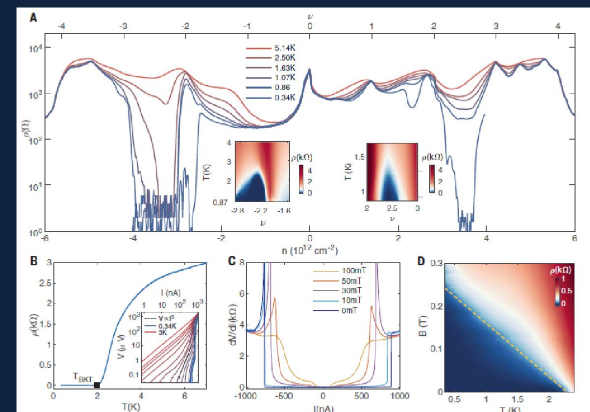


SUPERCONDUCTIVITY

Electric field–tunable superconductivity in alternating-twist magic-angle trilayer graphene

Zeyu Hao[1*], A. M. Zimmerman[1*], Patrick Ledwith[1], Eslam Khalaf[1], Danial Haie Najafabadi[1], Kenji Watanabe[2], Takashi Taniguchi[3], Ashvin Vishwanath[1], Philip Kim[1†]

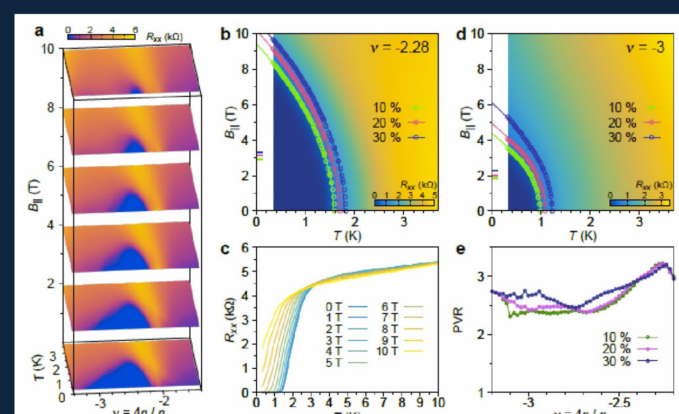
Hao et al., Science 371, 1133–1138 (2021) 12 March 2021



Large Pauli Limit Violation and Reentrant Superconductivity in Magic-Angle Twisted Trilayer Graphene

Yuan Cao,[1,*,†] Jeong Min Park,[1,*,†] Kenji Watanabe,[2] Takashi Taniguchi,[2] and Pablo Jarillo-Herrero[1,†]

arXiv:2103.12083



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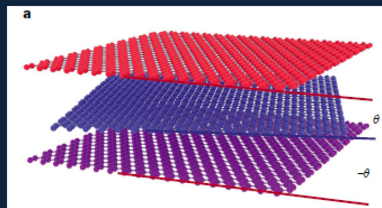

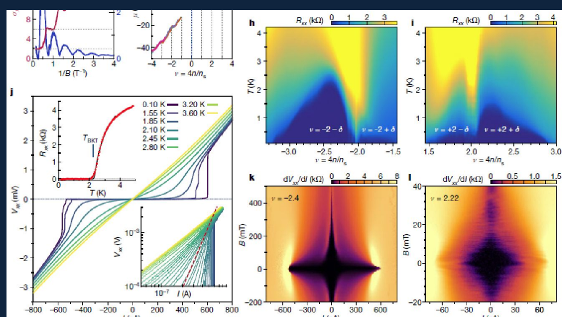

SUPERCONDUCTIVITY

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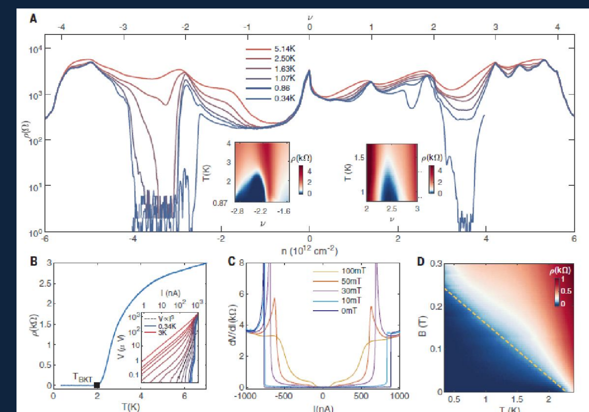

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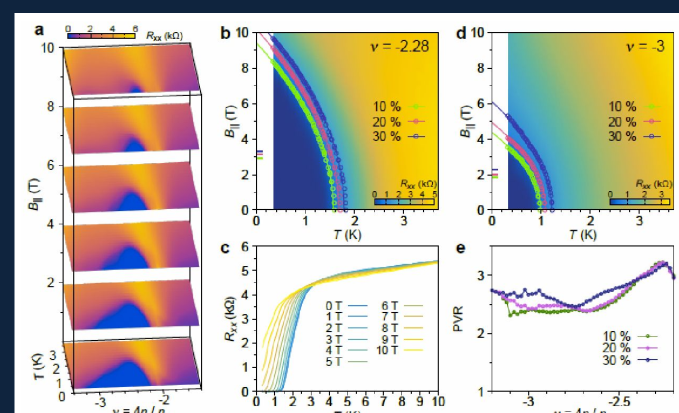

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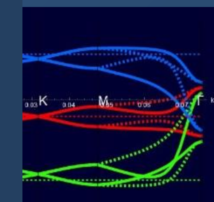
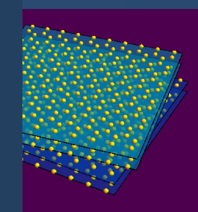
$10 - 40\ \text{meV}$

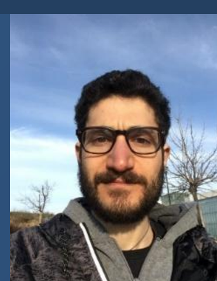




- Twisted bilayer graphene is a unique material
- There is a large number of open problems

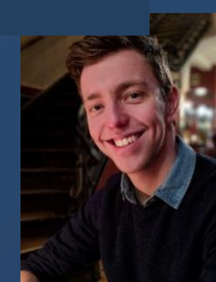
hopping

Niels R. Walet



Tomasso Cea

Pierre Pantaleon



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